Surface Waves in Turbulent and Laminar Submesoscale Flow

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with Peter Hamlington (CU-Boulder), Luke Van Roekel (Northland College),
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Peter Sullivan (NCAR), Jim McWilliams (UCLA), Mark Hemer (CSIRO)

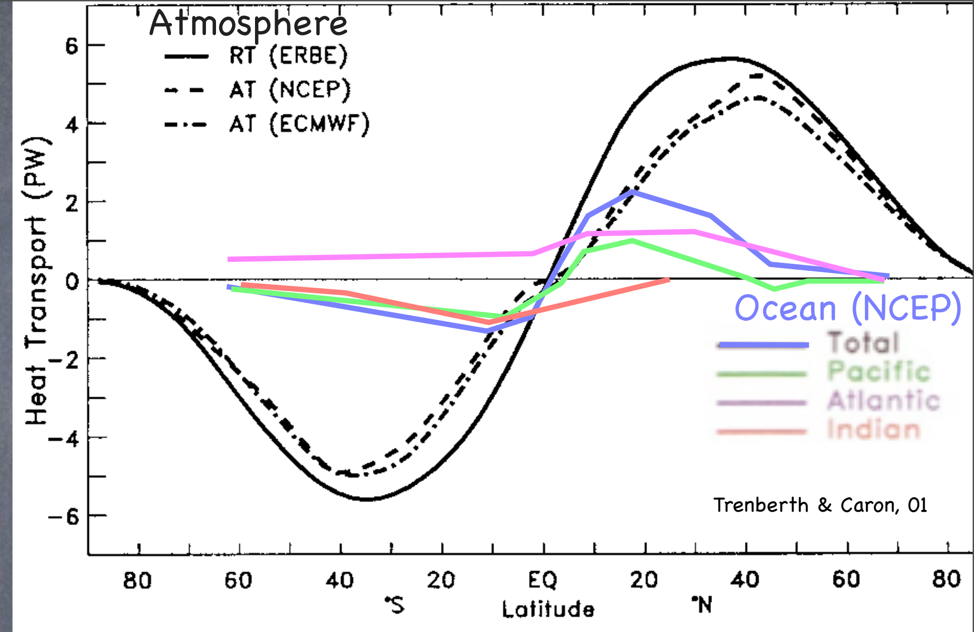
Boulder Fluids Seminar

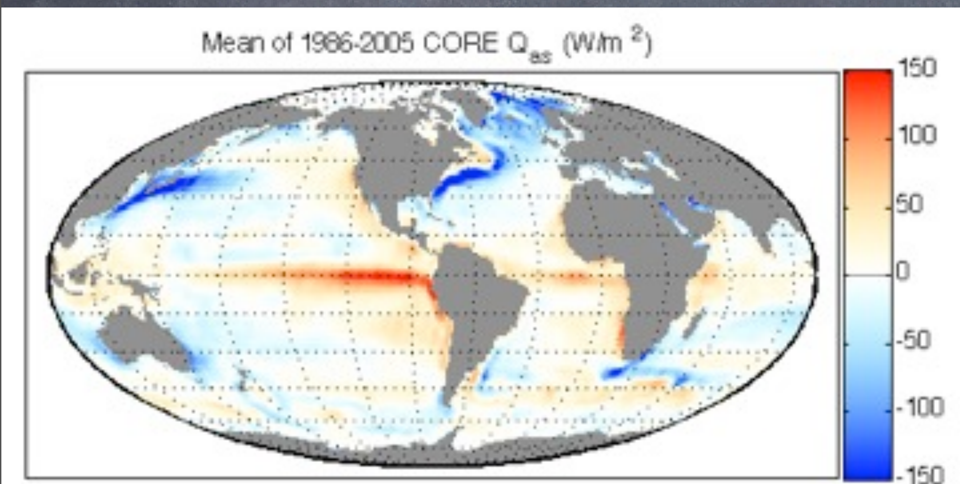
Sponsors: NSF 1245944, 0934737, 0825614, NASA NNX09AF38G

Air-Sea Flux Errors vs. Data

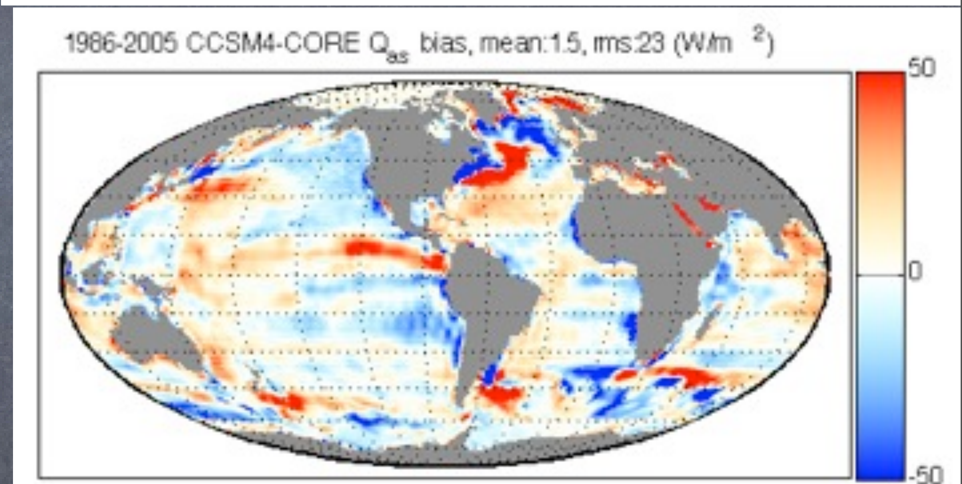
Heat capacity & mode of transport is different in A vs. O

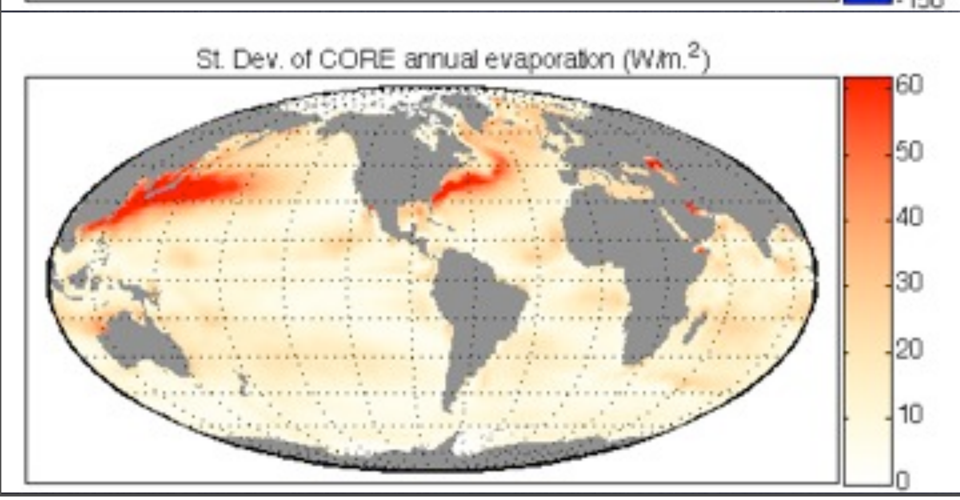
S. C. Bates, BFK, S. R. Jayne, W. G. Large, S. Stevenson, and S. G. Yeager. Mean biases, variability, and trends in air-sea fluxes and SST in the CCSM4.Journal of Climate, 25(22):7781-7801, 2012.



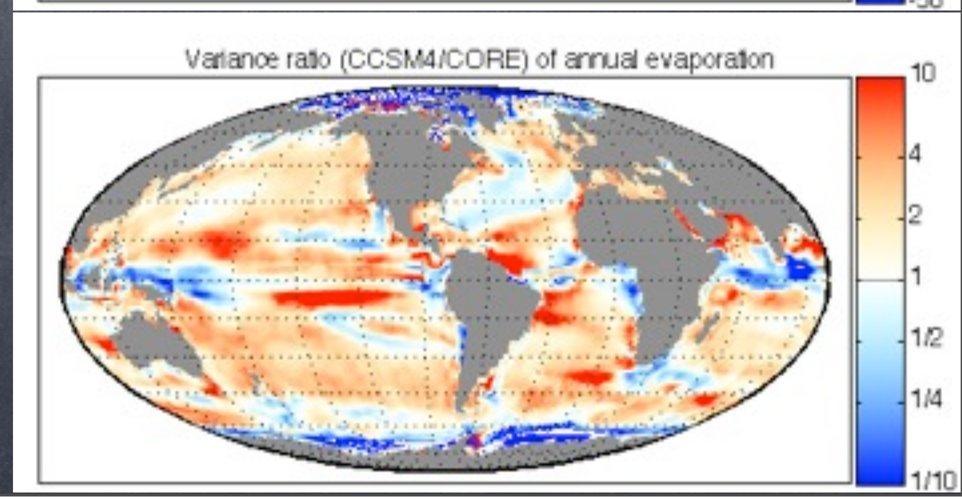


Mean





Annual 9-15mo



With nearly incompressible (small density variations) approximation & approximated rotating Earth:

A simple(?) set of 5 vars

Summary of Boussinesq Equations

$$\frac{D?}{Dt} \equiv \frac{\partial ?}{\partial t} + \mathbf{v} \cdot \nabla ?$$

The simple Boussinesq equations are, for an inviscid fluid:

momentum equations: $$\frac{D\boldsymbol{v}}{Dt} + \boldsymbol{f} \times \boldsymbol{v} = -\nabla \phi + b\mathbf{k}, \qquad \text{(B.1)}$$

mass conservation: $$\nabla \cdot \boldsymbol{v} = 0, \qquad \text{(B.2)}$$

buoyancy equation: $$\frac{Db}{Dt} = \dot{b}. \qquad \text{(B.3)}$$

Vallis, 06

If you want, it's easy to distinguish buoyancy into contributions from Temperature and from Salinity (since we are near surface--linear EOS is OK)

Geostrophy, Hydrostasy, & Thermal Wind

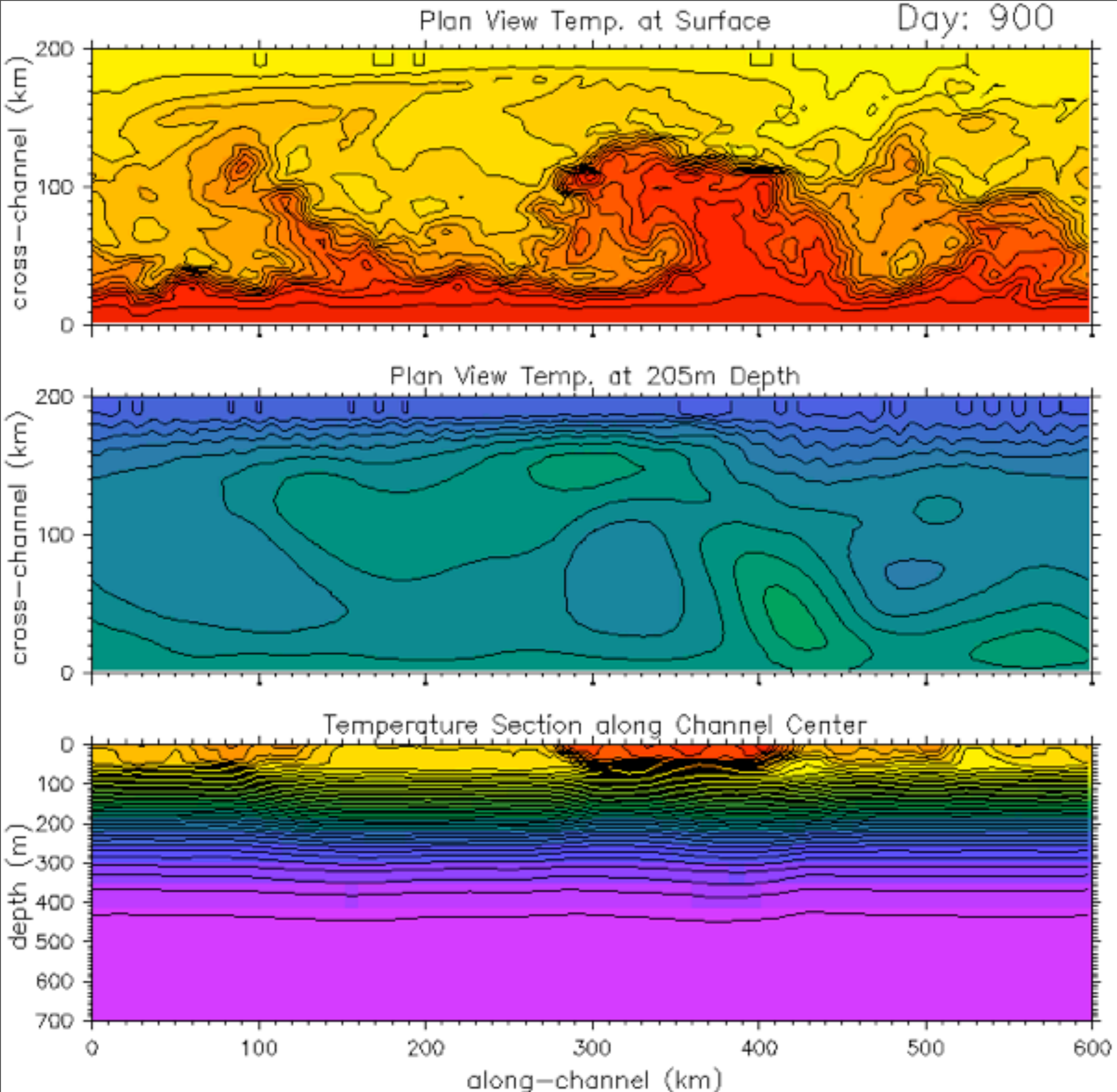
Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:

Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin (L>>H)

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Adding forcing (air-sea) and advection of buoyancy by this flow--you have (nearly) all large-scale ocean physics!



Big, Deep (mesoscale)

interact with

Little, Shallow (submeso)

BFK, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6): 1145-1165, 2008.

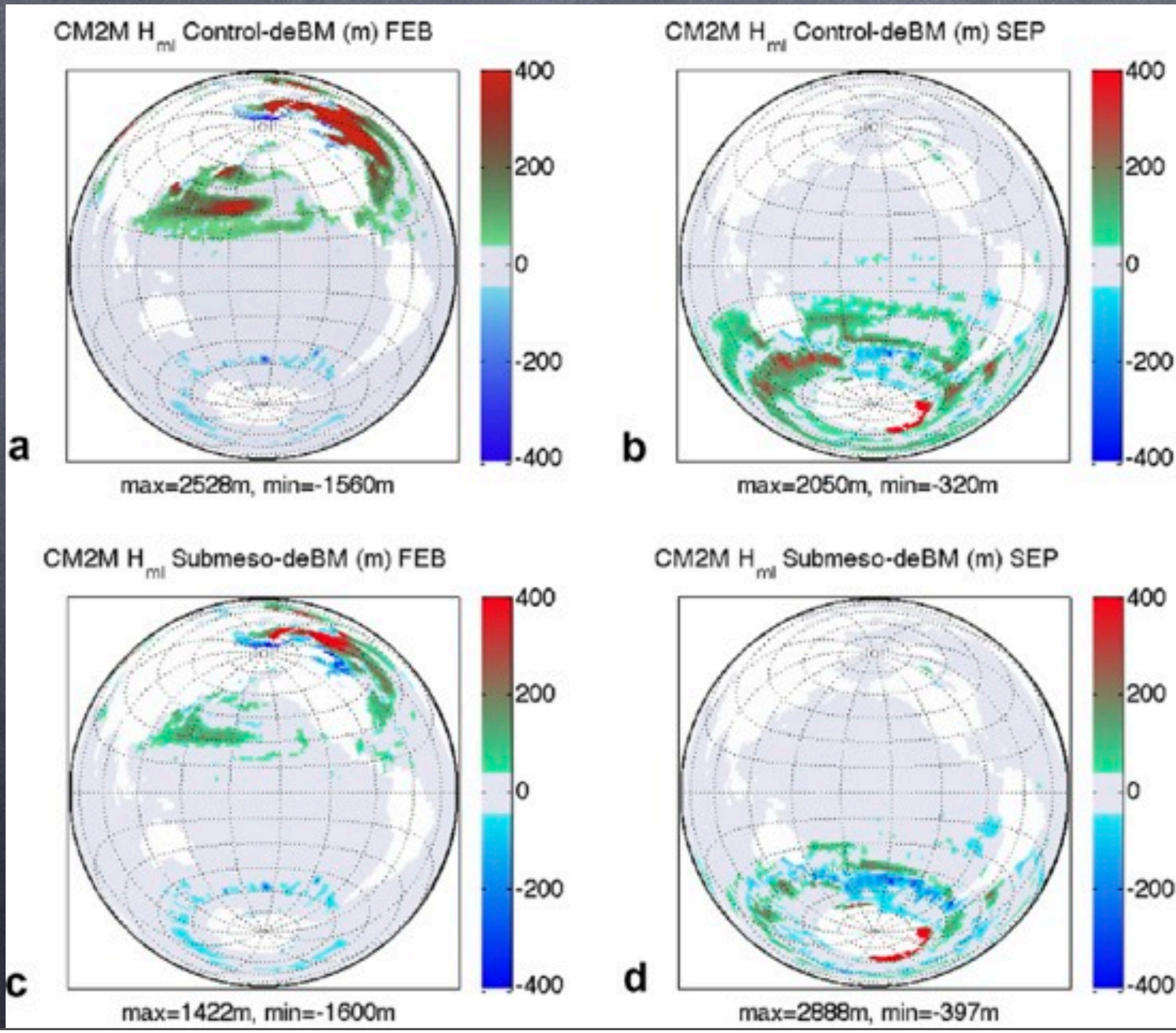
Climate affected by (Submeso) Mixed Layer Eddy Restratification

Affects AMOC, Sea Ice, SST, SSS, CFCs, Water Masses, Air-Sea, etc.

February | September

Error w/o MLE

Error with MLE



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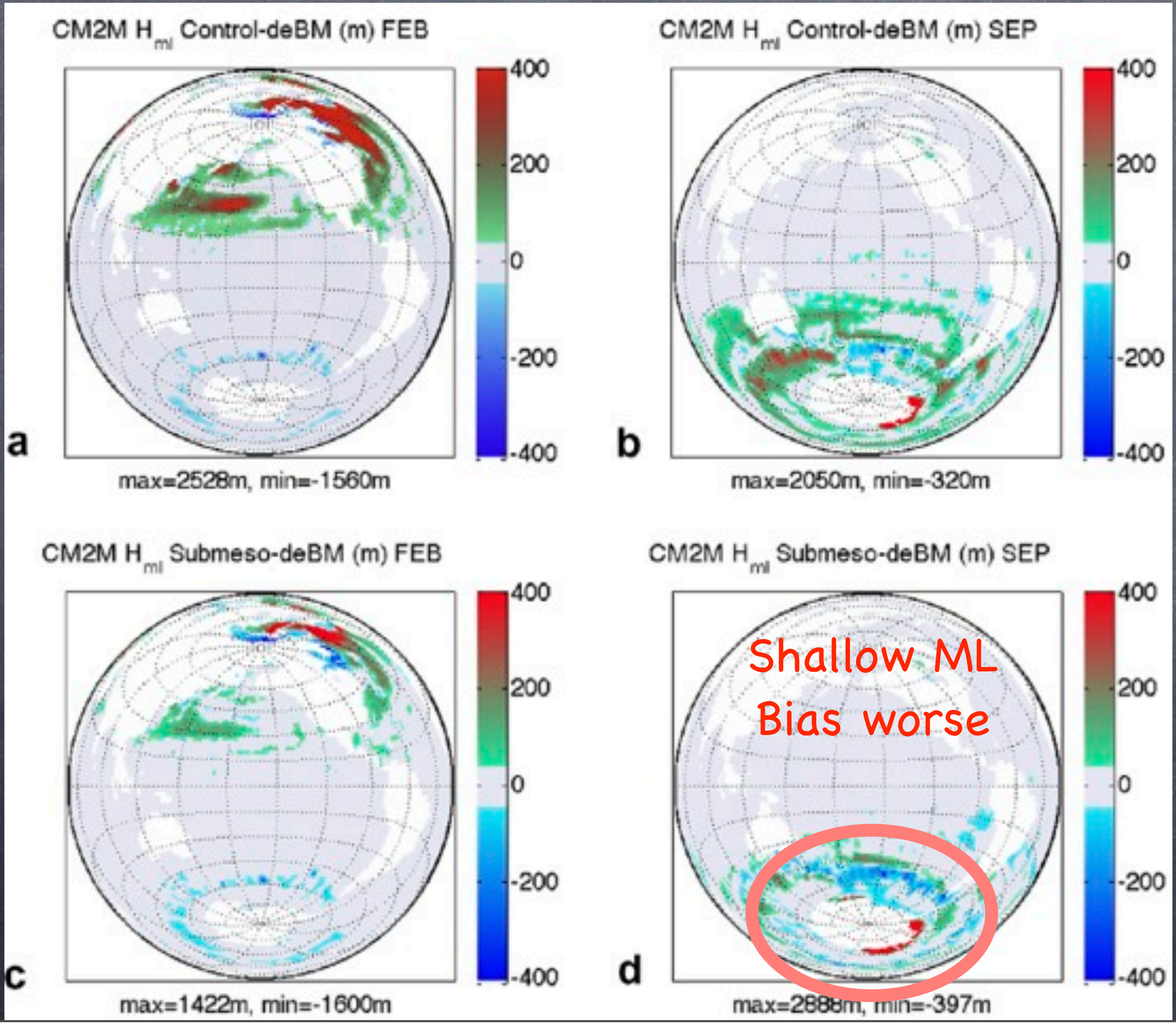

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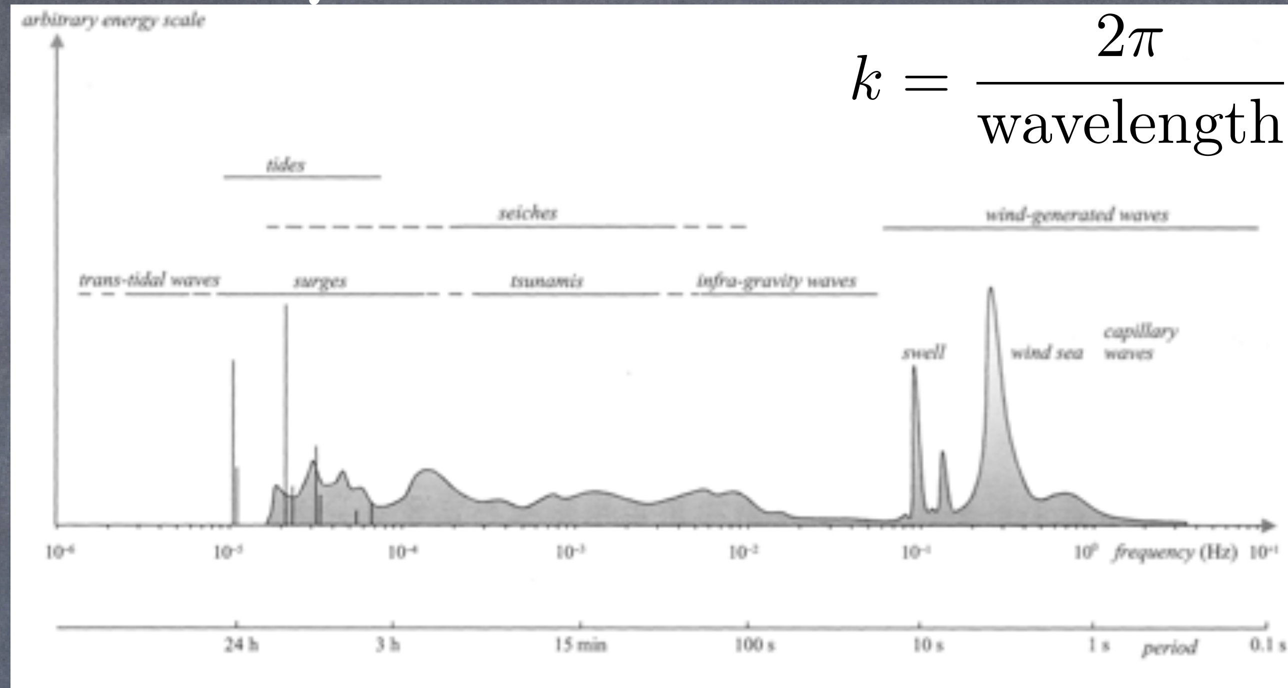
- Method: Study a small-scale phenomenon (100m-10km submeso mixed layer fronts & eddies), parameterize, assess impact globally, and improve climate models
    - In submeso, we relied heavily on thermal wind
- Problem with models: they are only slightly smarter than we are (they don't do what we don't put in!)
- But, what about the effects of things that aren't geostrophic & hydrostatic?
    - For example, waves and near-surface 3d turbulence

Ageostrophic, Nonhydrostatic Waves

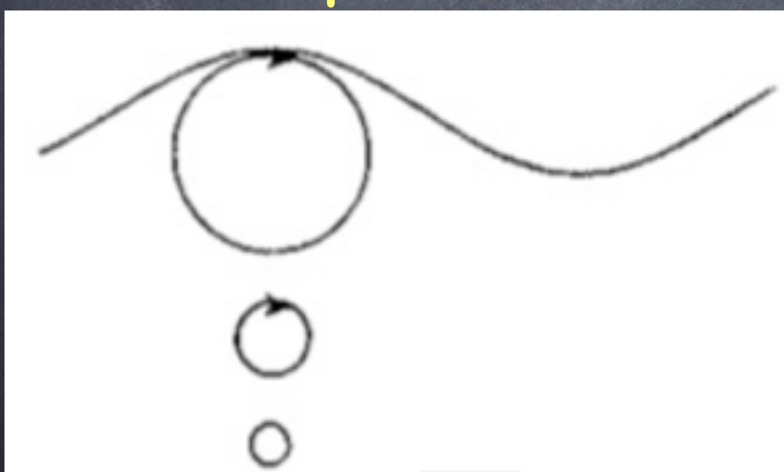
Look for fast, small solutions of the free-surface Boussinesq Equations

Linearize for not steep waves



$$k = \frac{2\pi}{\text{wavelength}}$$

Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)



$$\omega = \sqrt{gk}$$

$$c_p = 2c_g = \sqrt{g/k}$$

Craik-Leibovich Boussinesq

- Formally a multiscale asymptotic equation set:
  - 3 classes: Small, Fast; Large, Fast; Large, Slow
  - Solve first 2 types of motion in the case of limited slope (ka), irrotational --> Deep Water Waves!
  - Must also assume slowly-varying wave packets
  - Average over deep water waves in space & time,
  - Arrive at Large, Slow equation set:

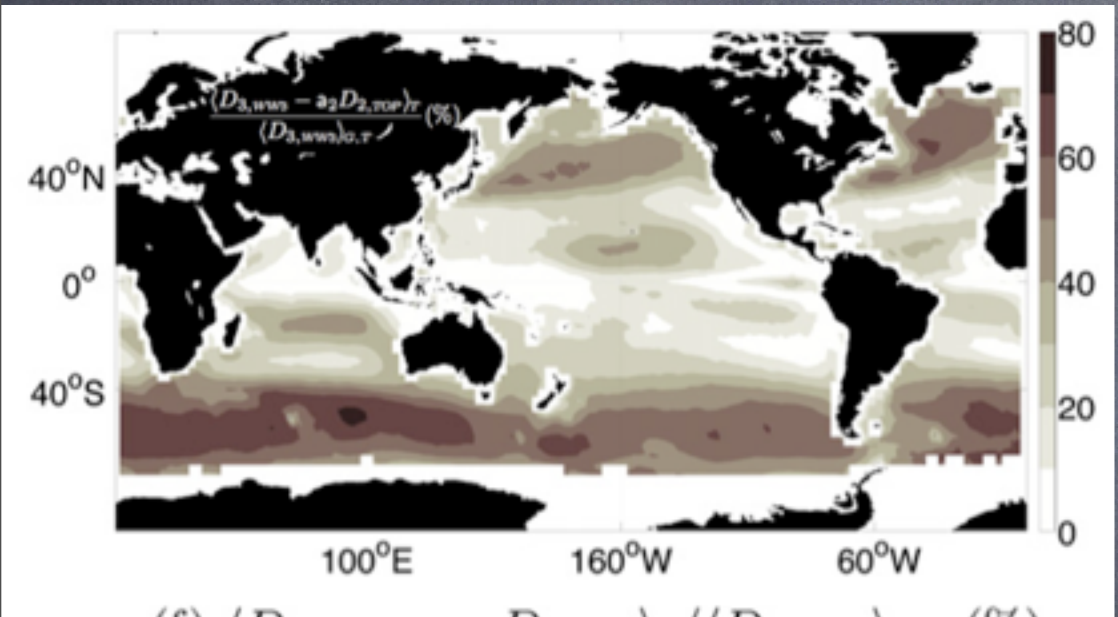
$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^\dagger + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

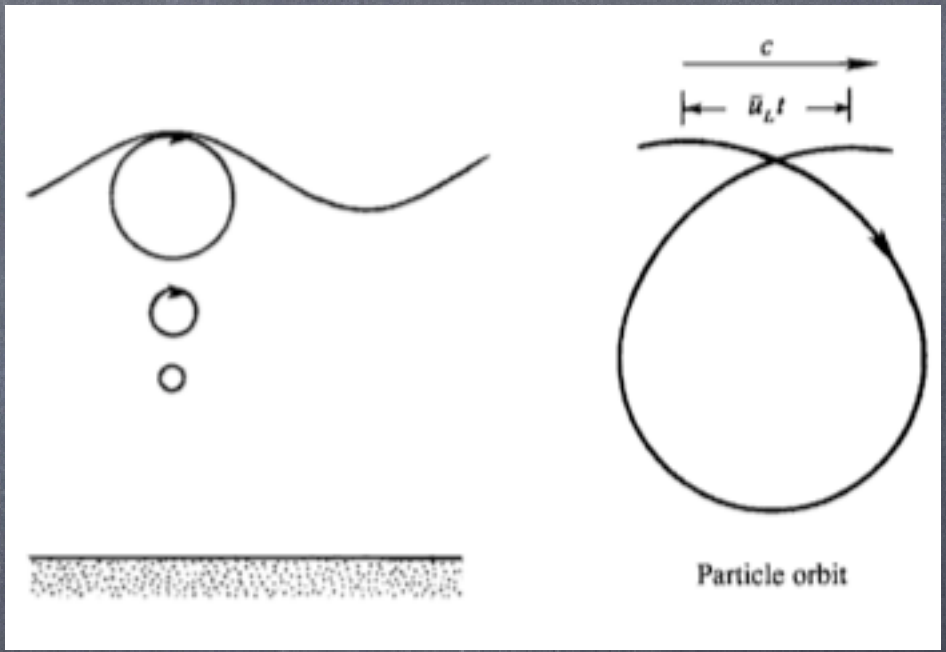
$$\mathbf{v}_s = \text{Stokes Drift}$$

Craik & Leibovich 1976; Gjaja & Holm 1996; McWilliams *et al.* 2004

How well do we know Stokes Drift? <50% discrepancy



(f) $\langle D_{3,WW3} - a_2 D_{2,TOP} \rangle_T / \langle D_{3,WW3} \rangle_{G,T}$ (%)



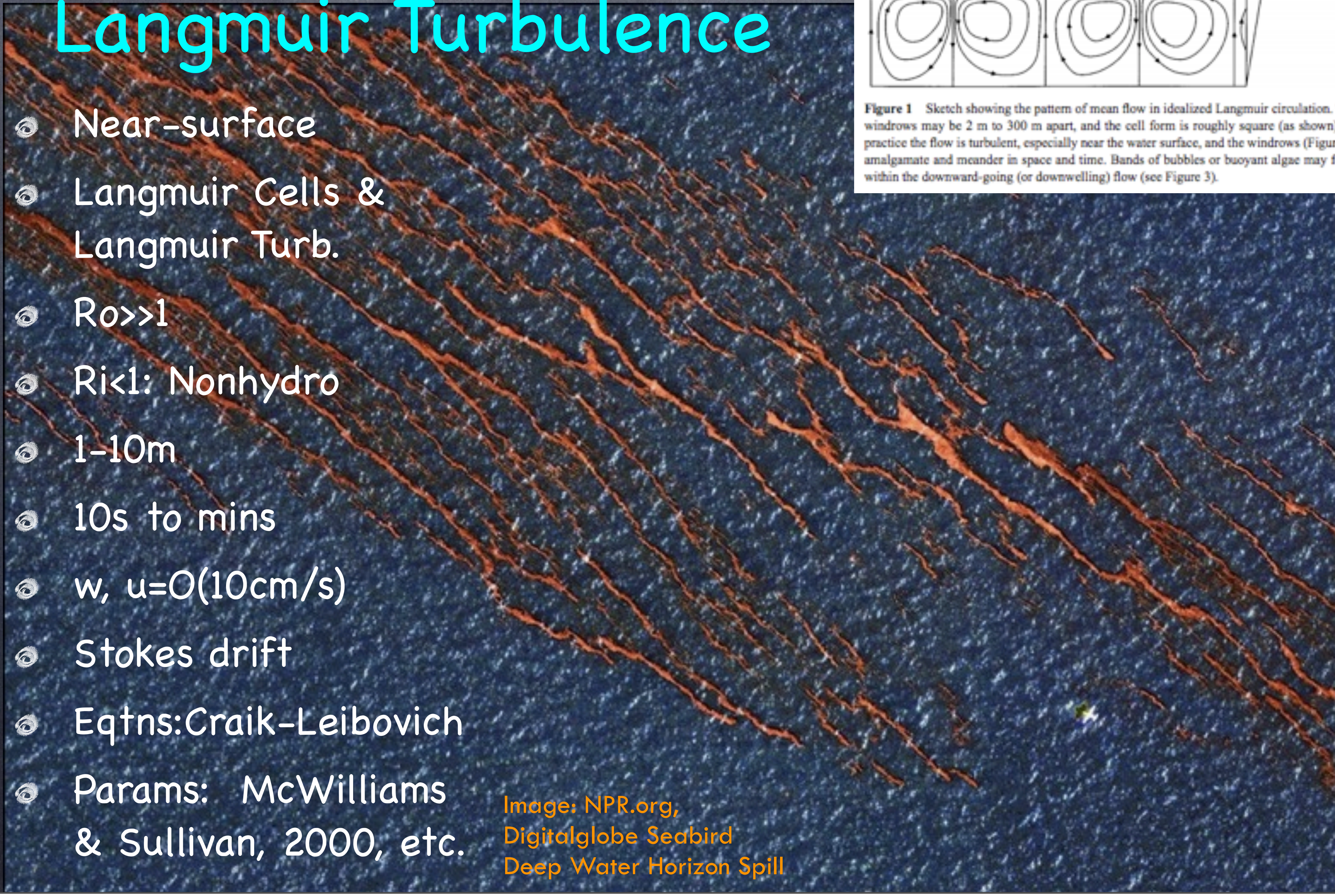
RMS error in measures of surface Stokes drift, between wave models (not shown) or model vs. altimeter (shown)

A. Webb and BFK. Wave spectral moments and Stokes drift estimation. *Ocean Modelling*, 40(3-4):273-288, 2011.

Now, we've got the CLB equations & estimated global Stokes, what to do?

- 1) Stokes-driven small-scale turbulence (Large Eddy Simulations of CLB)
- 2) Laminar submesoscale flow with Stokes Coriolis & Stokes Vortex forces (Analytic Solns of CLB)
- 3) Wave-driven turbulence interacting with submesoscale flow (Multiscale LES of CLB)

The Character of the Langmuir Turbulence



- Near-surface
- Langmuir Cells & Langmuir Turb.
- Ro>>1
- Ri<1: Nonhydro
- 1-10m
- 10s to mins
- w, u=O(10cm/s)
- Stokes drift
- Eqtns:Craik-Leibovich
- Params: McWilliams & Sullivan, 2000, etc.

Image: NPR.org, Digitalglobe Seabird Deep Water Horizon Spill

image: Thorpe, 04

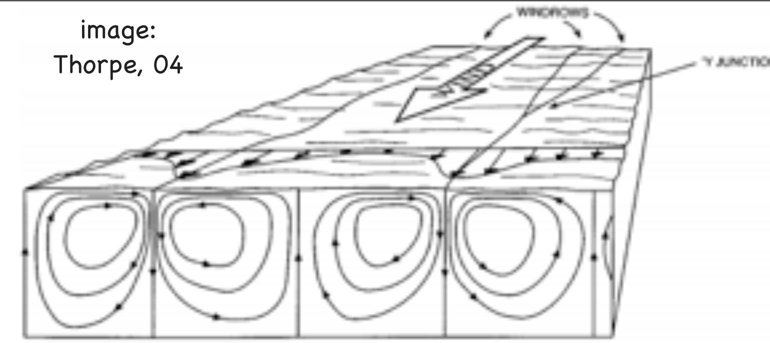
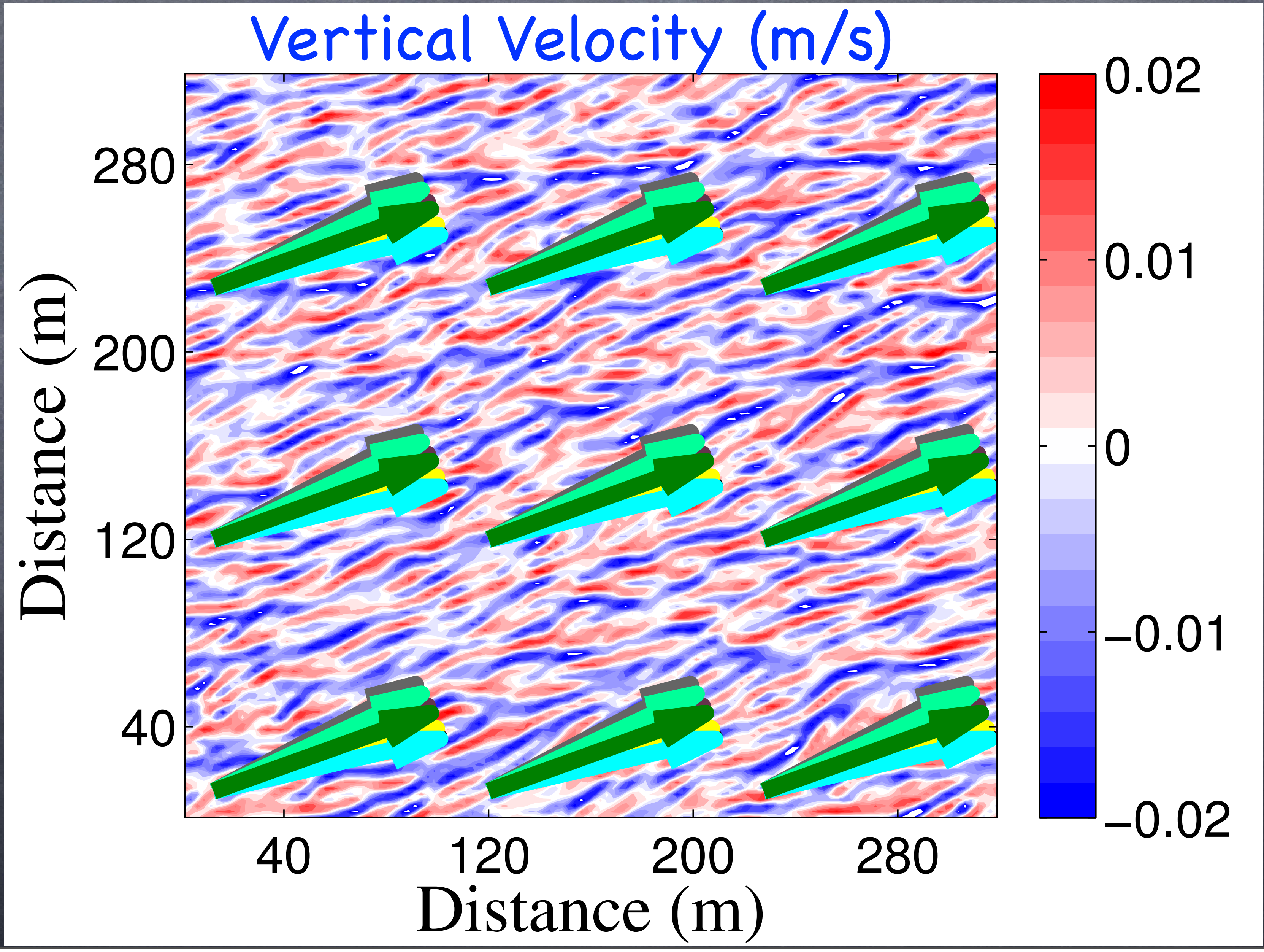


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. Th windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). I practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2 amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

CLB as equations for Large Eddy Simulations:
Interesting in Data: Misaligned Wind & Waves



L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, May 2012.

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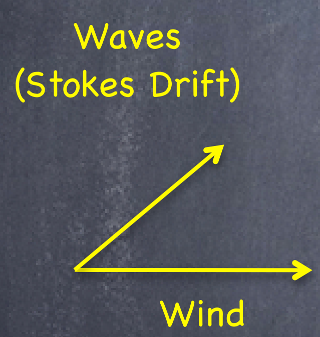

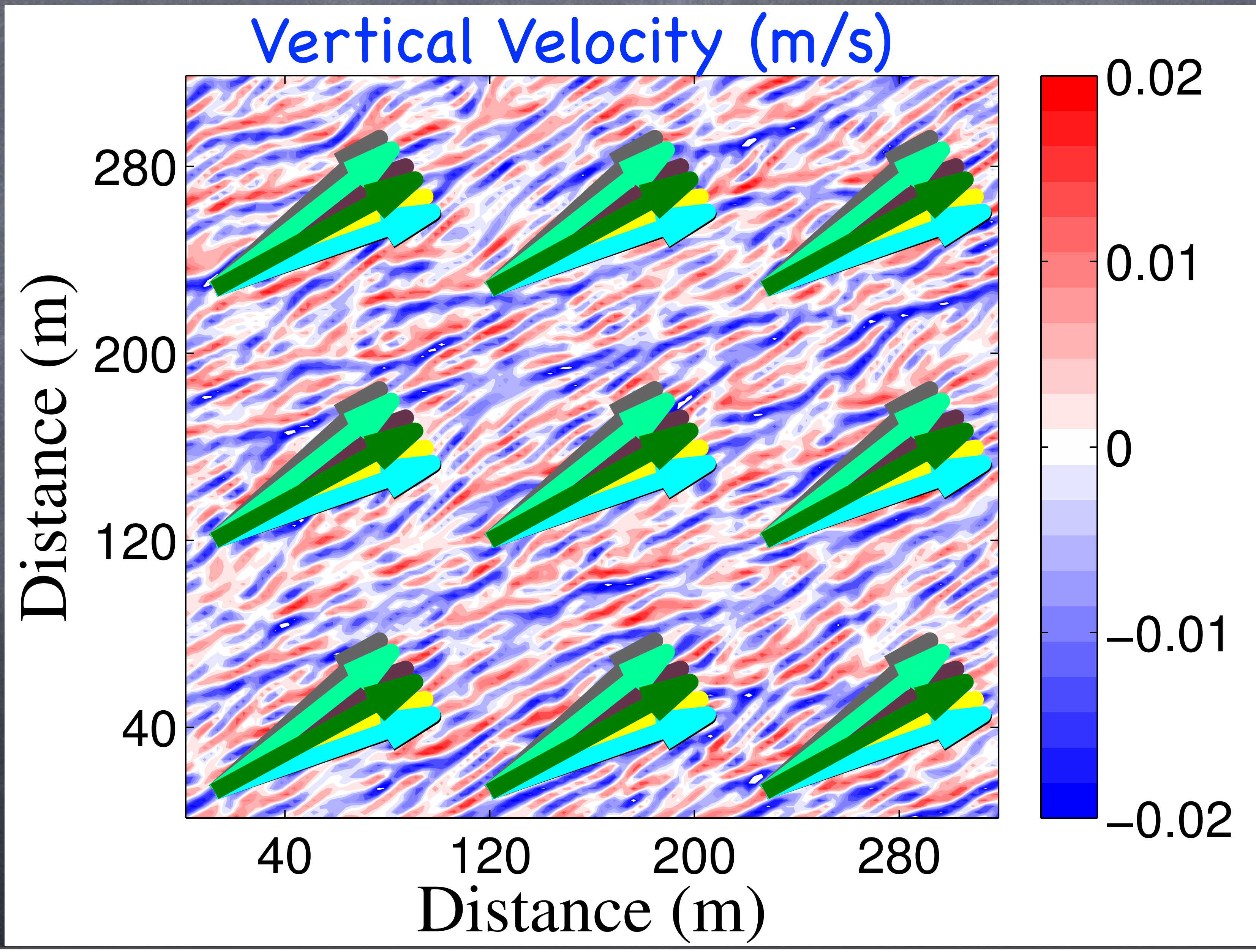

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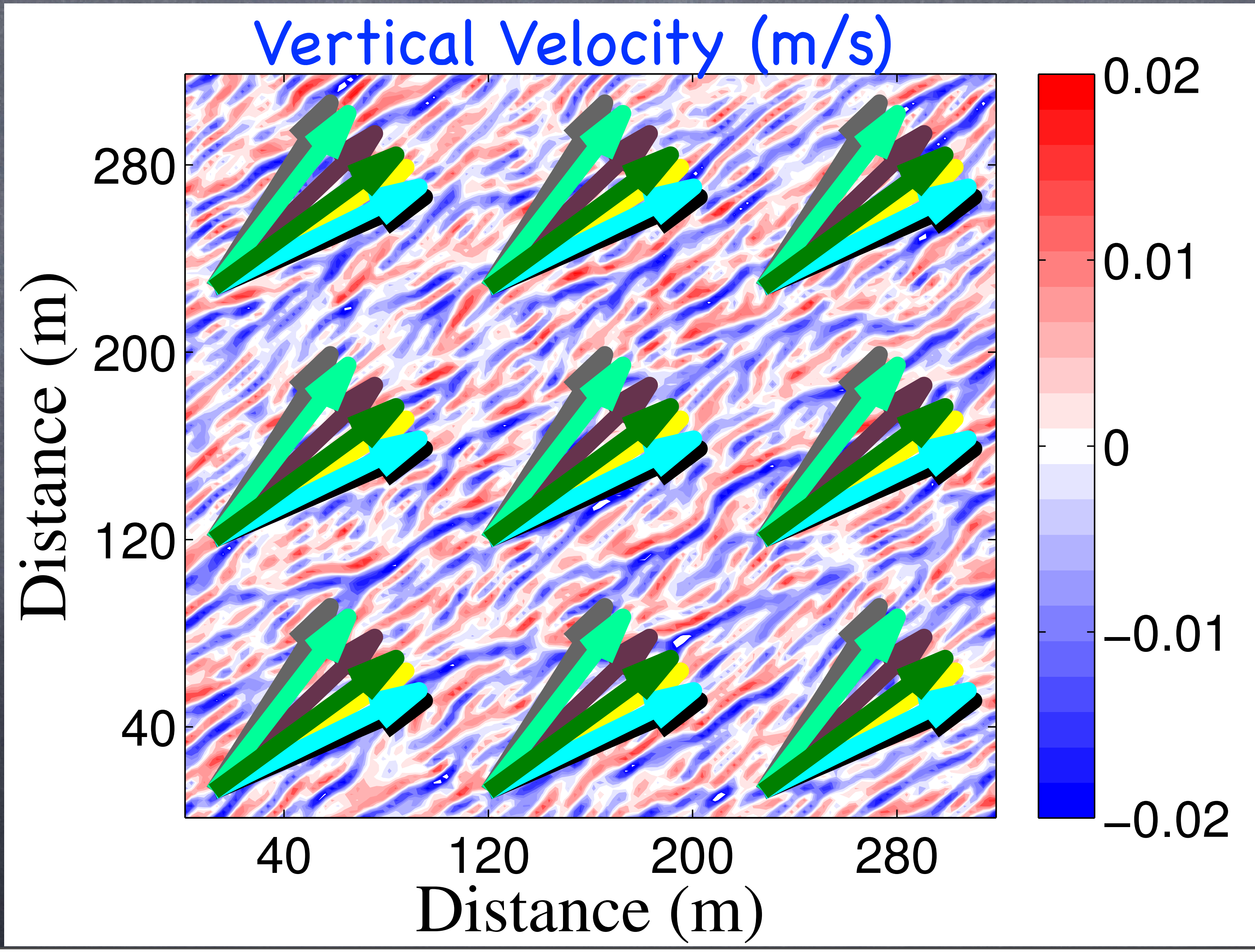

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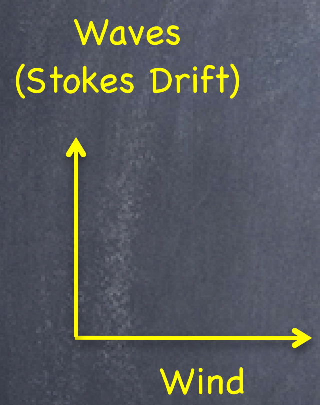

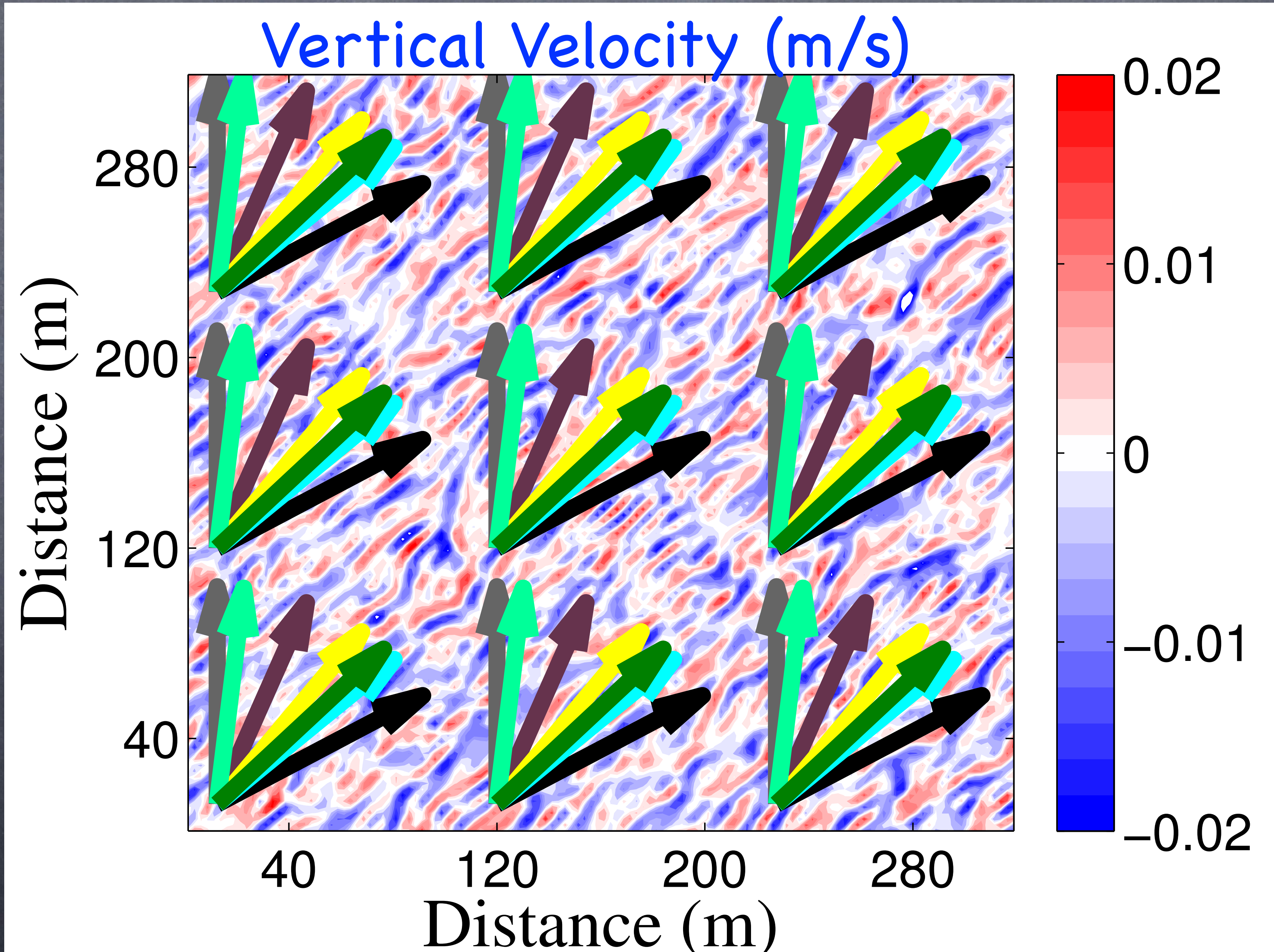

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Why? Vortex Tilting Mechanism

In CLB: Tilting & Stretching occur in direction of Lagrangian shear :

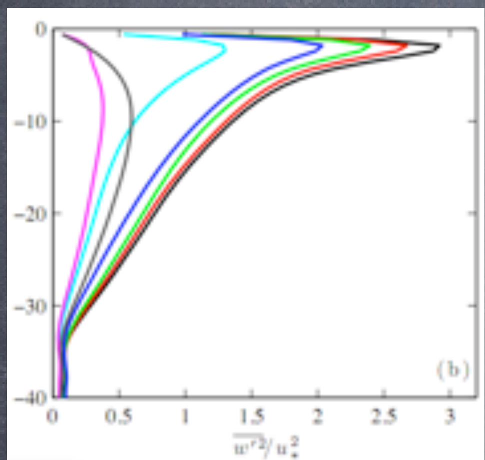
$$\frac{\partial \omega}{\partial t} + \underbrace{(\mathbf{v}_L \cdot \nabla)\,\omega}_{AD} = \underbrace{([\omega + f\hat{\mathbf{z}}] \cdot \nabla)\,\mathbf{v}_L}_{TS} + \underbrace{\nabla b \times \hat{\mathbf{z}}}_{BT}$$

$$\mathbf{v}_L \equiv \mathbf{v} + \mathbf{v}_s, \qquad \omega \equiv \nabla \times \mathbf{v}$$

rescaling by projection collapses LES results!

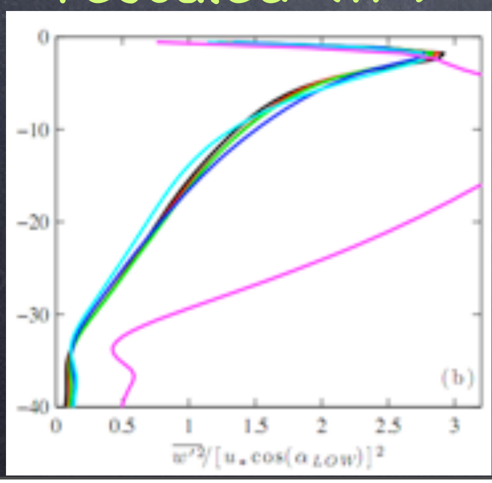
<w^2>

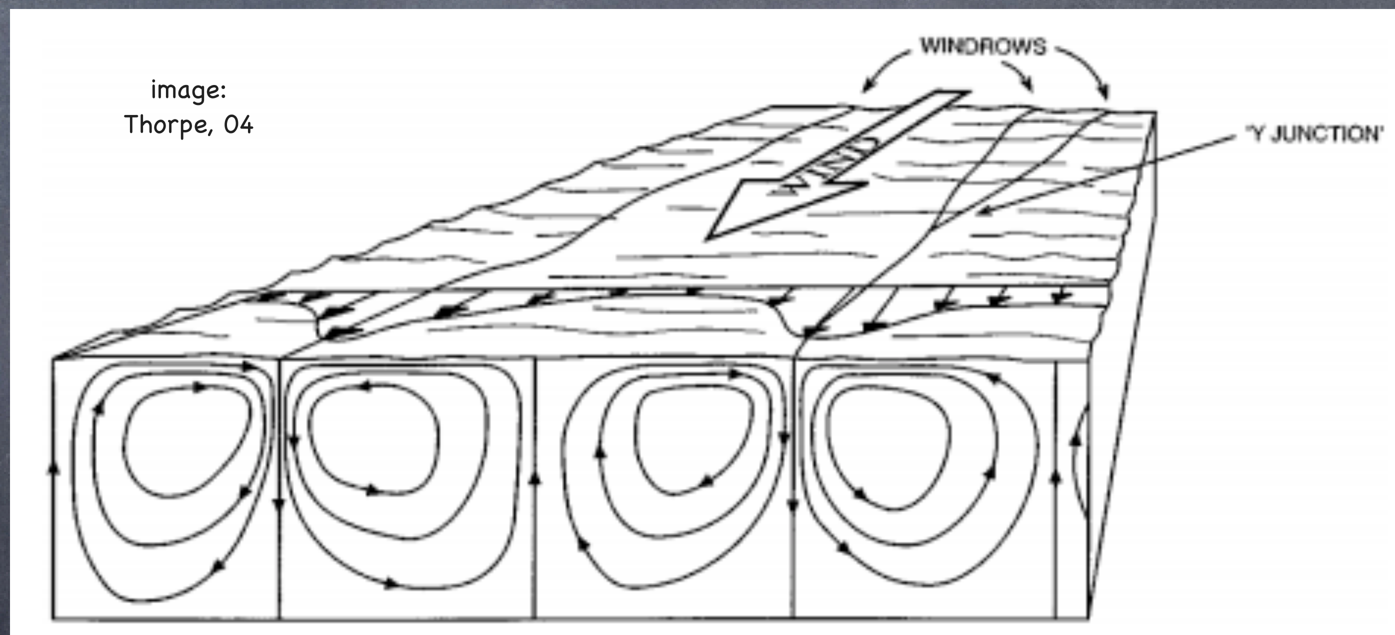
depth



rescaled <w^2>

depth





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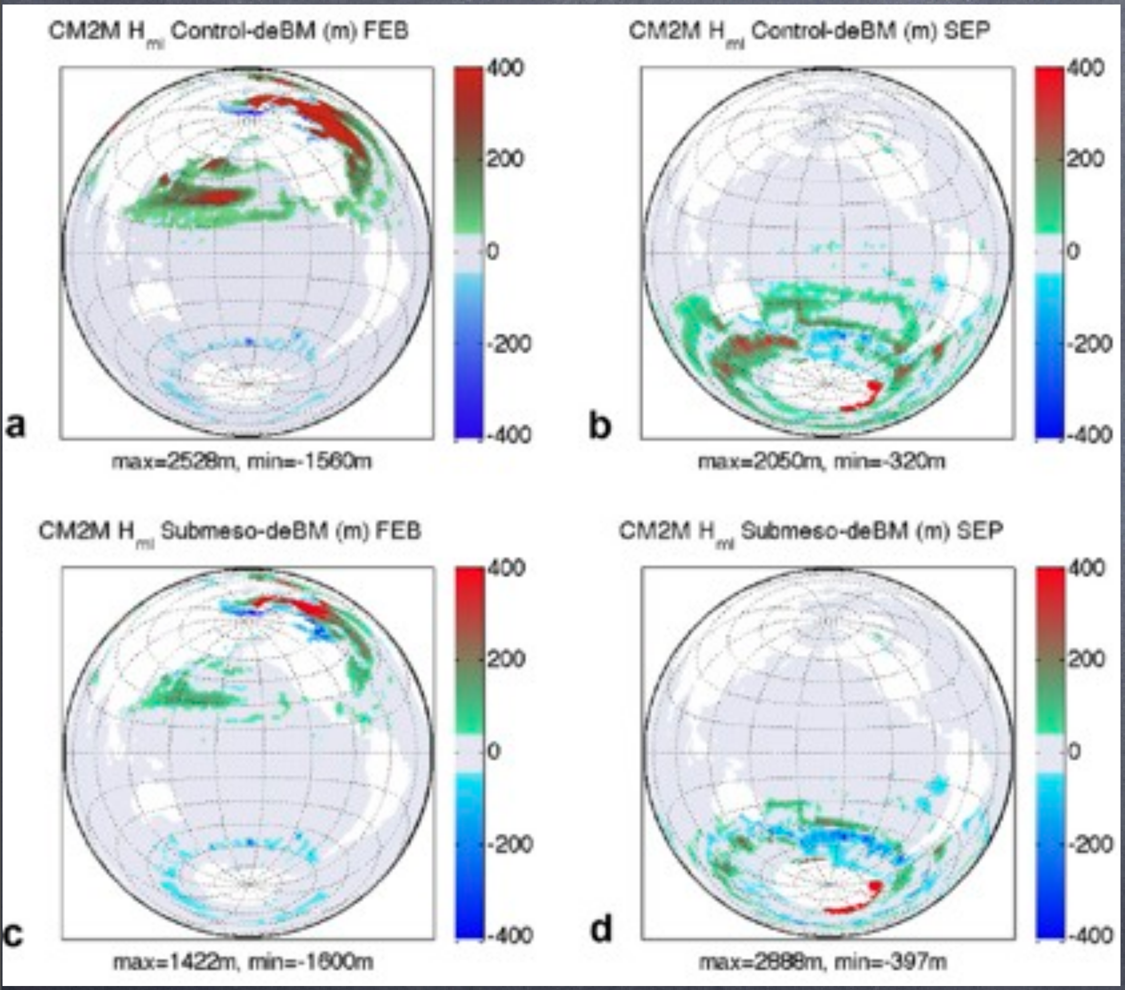
Recall our problem with the (submeso) Mixed Layer Eddy Restratification--Southern Ocean too shallow!



Bias w/o MLE

Sallee et al. (2013) have shown that a too shallow S. Ocean MLD is true of most* present climate models

salinity forcing or ocean physics?

*true for CMIP5 multi-model ensemble

BFK, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

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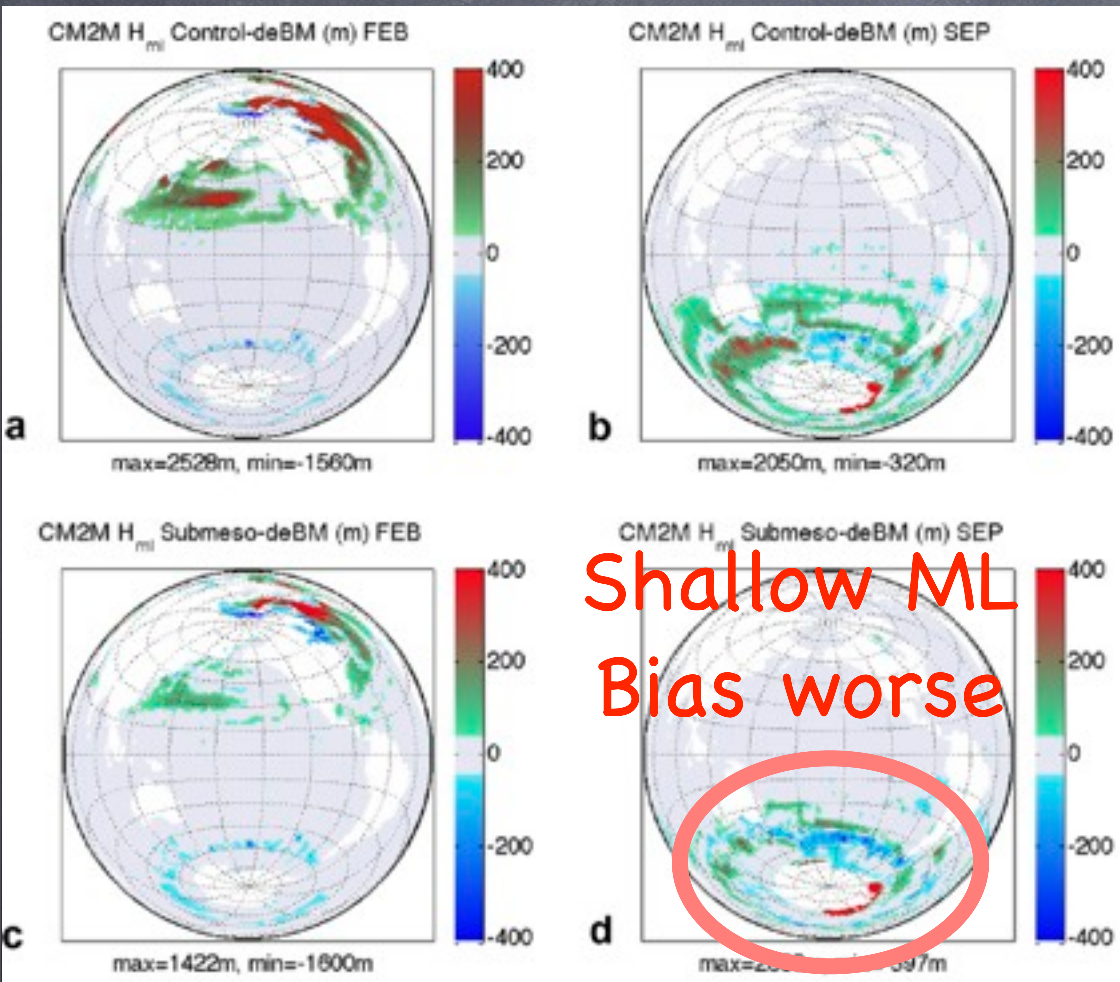

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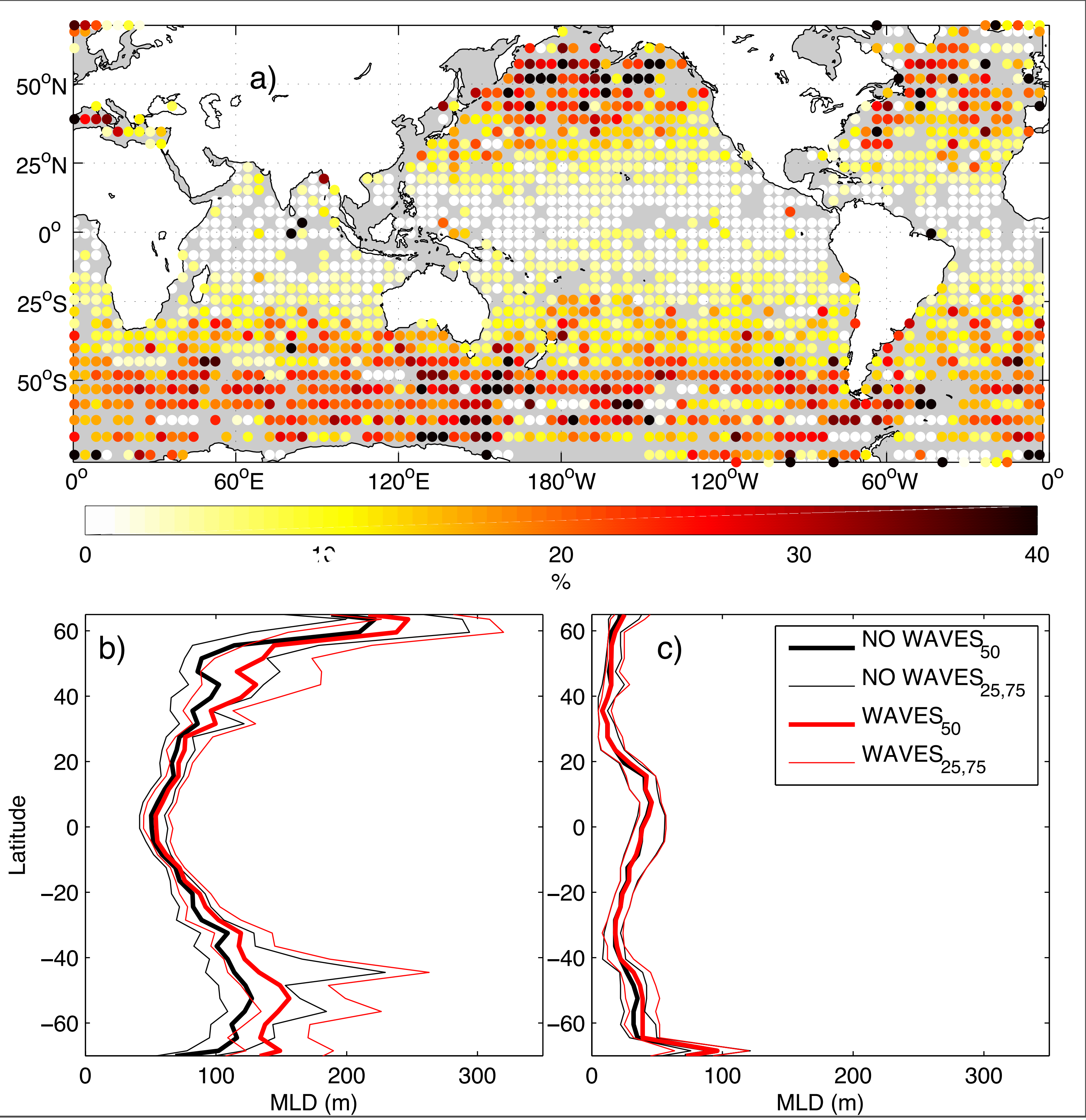
Including Wave-driven Mixing

(Harcourt 2013 parameterization shown)

S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, BFK, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. Geophysical Research Letters, 39(18):L18605, 9pp, 2012.

M. A. Hemer, BFK, & R. R. Harcourt. Quantifying the effects of wind waves the the coupled climate system, in prep. 2013.



So, Waves can Drive turbulence that affects large scale:

What about direct effects of waves on larger scales?

Stokes Coriolis & Stokes Vortex Forces on Submesoscales

Recall, Subinertial Boussinesq Equations Dominated by:

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

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Stokes Coriolis & Stokes Vortex Forces on Submesoscales

Craik-Leibovich Boussinesq Subinertial Dominated By:

(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the buoyancy gradients govern the Lagrangian flow, not the not the Eulerian!

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Craik-Leibovich Boussinesq Subinertial Dominated By:

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Buoyancy & PV also advected by Lagrangian Flow!

All GFD is for the Lagrangian Flow??

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Can we just forget the whole thing and interpret large scales as Lagrangian velocities?

$$[\mathbf{f} + \nabla \times \mathbf{v}] \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = -\nabla b$$

No, because vortex force is different!

The "Rossby #" for waves, is big *more often* than Ro is

Talk to Haney for more!!!

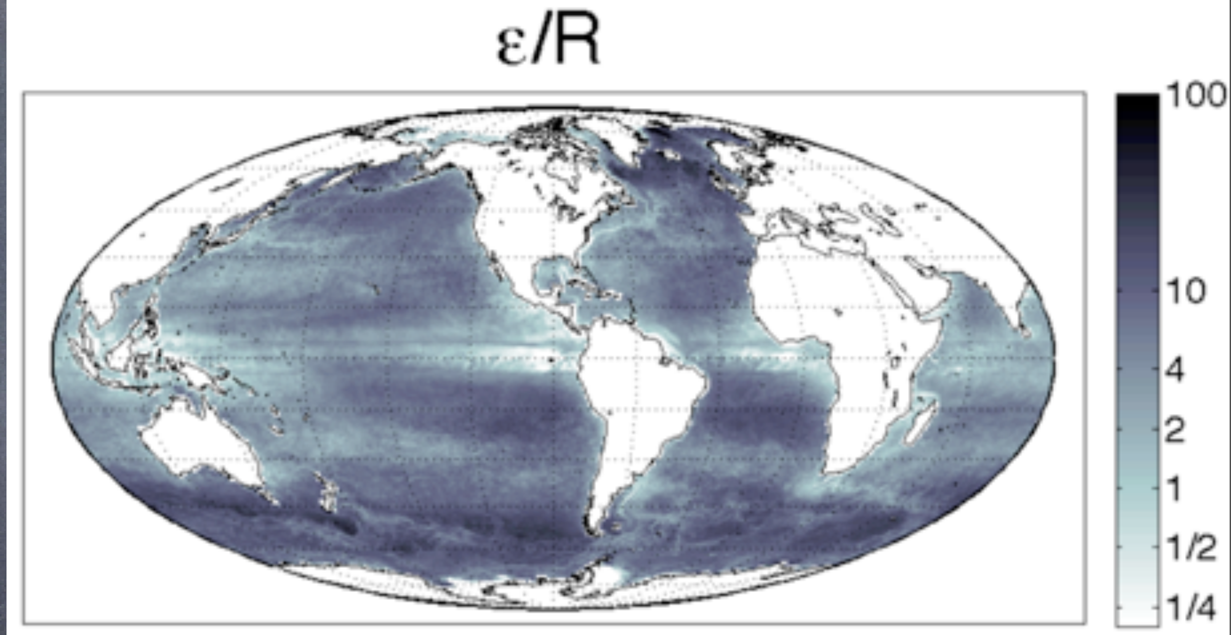
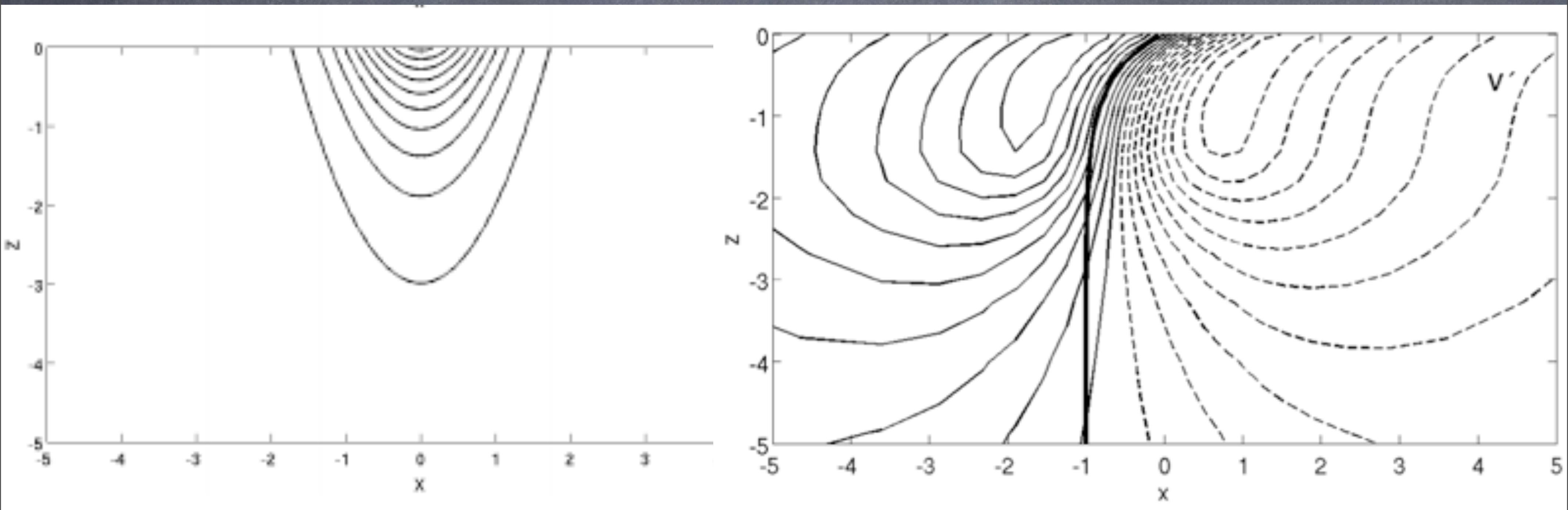


FIGURE 1. Estimated ratio $\epsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}| h)/(|\mathbf{u}|^2 h_s)$ governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity ($\mathbf{u}$) is taken as the AVISO weekly satellite geostrophic velocity or $-\mathbf{u}_s$ (for anti-Stokes flow) if $|\mathbf{u}_s| > |\mathbf{u}|$. The front/filament depth (h) is estimated as the mixed layer depth from the de Boyer Montégut *et al.* (2004) climatology. An exponential fit to the Stokes drift of the upper 9m projected onto the AVISO geostrophic velocity provides $\mathbf{u}_s \cdot \mathbf{u}$ and h_s. Stokes drift is taken from the WaveWatch-3 simulation described in Webb & Fox-Kemper (2011). $\mathbf{u}$, $\mathbf{u}_s$, and h_s are all for the year 2000, while h is from a climatology of observations over 1961-2008. The year 2000 average of $\epsilon/\mathcal{R}$ is shown.

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Waves (Stokes Vortex Force) example of wave-balanced Submeso flow

$\epsilon = 2, \epsilon \gg \mathcal{R}$ Near the "sweet spot"



Initial Submeso Front

Contours: 0.1

Perturbation on that scale due to waves

Contours: 1.4

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Movie: P. Hamlington

Talk to him for more!!

What about Langmuir-Submeso Interactions?

Perform large eddy simulations (LES) of CLB with a submesoscale temperature front with winds-- with and without Stokes drift

$$\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \mathrm{SGS}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + f\hat{\mathbf{z}}) \times \mathbf{u}_L = -\nabla \pi - \frac{g\rho\hat{\mathbf{z}}}{\rho_0} + \mathrm{SGS}$$

Wave & Wind Dir.

Computational parameters:

Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

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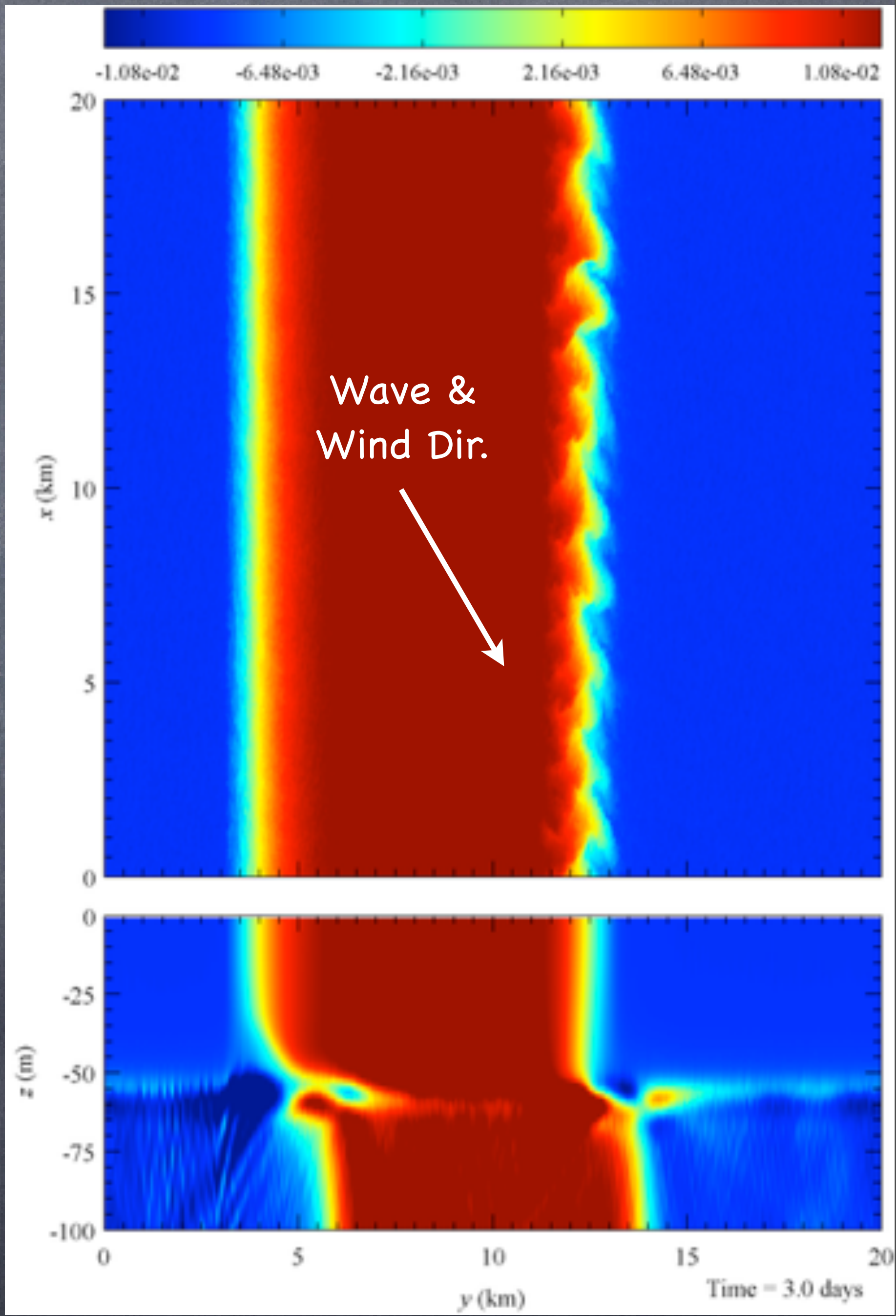
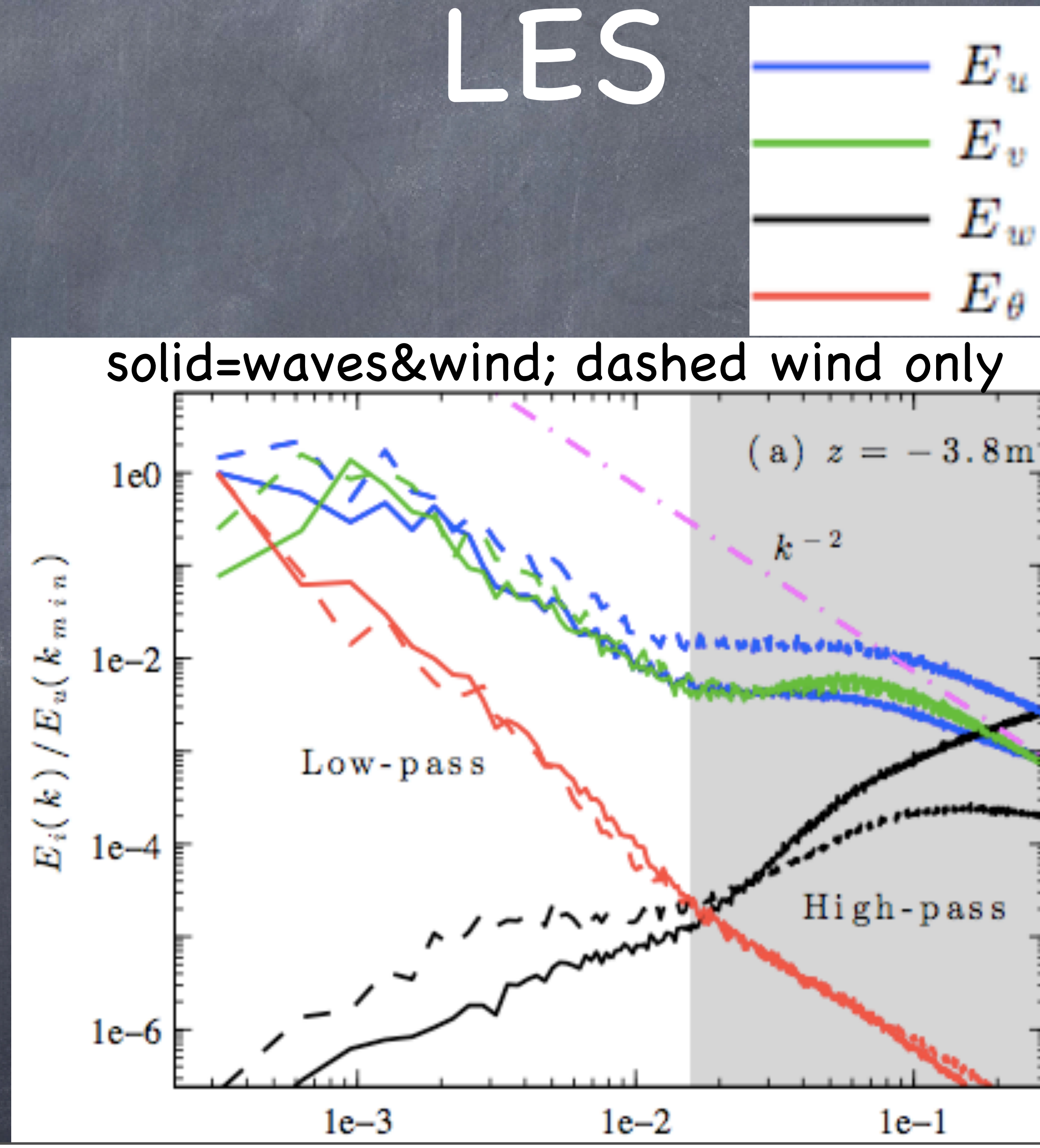


Overall results from multiscale LES

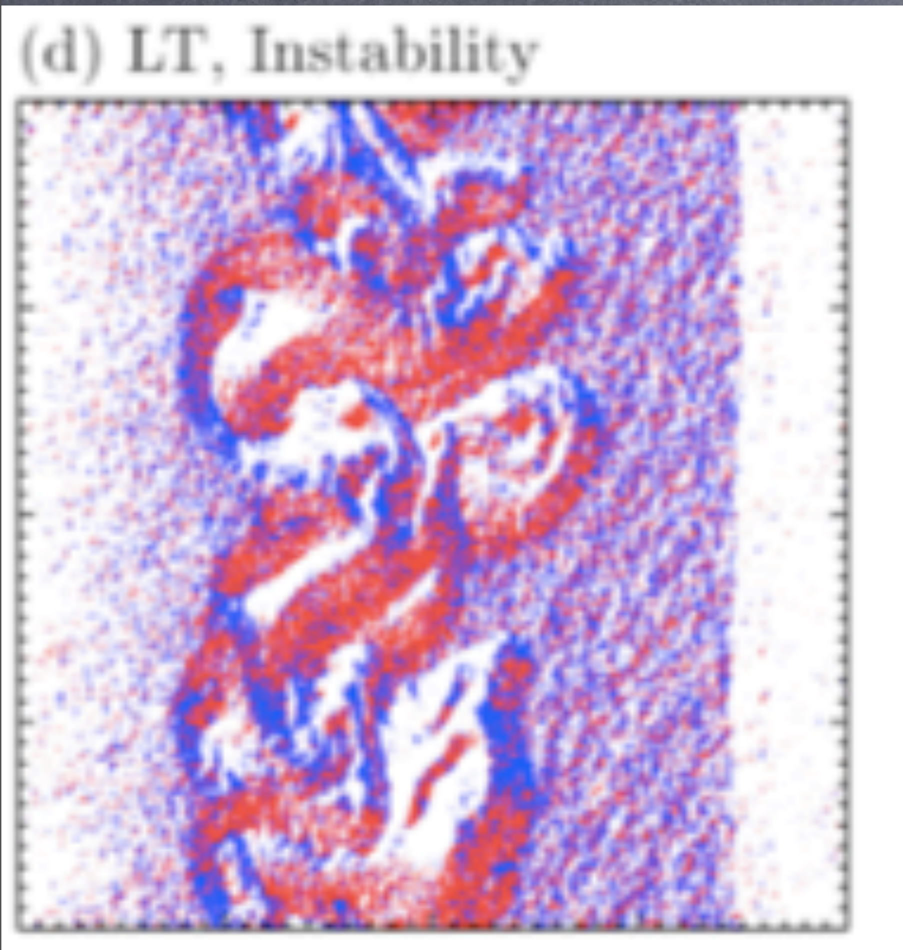
- Submesoscale flow is affected by wave-balance and enhanced <u'w'> (weaker surf. w/ Stokes)
- Strong two-way turbulent interactions are rare for this configuration
- Two turbulent cascades.
- Presence of waves greatly changes small scale from symmetric instability to gravitational

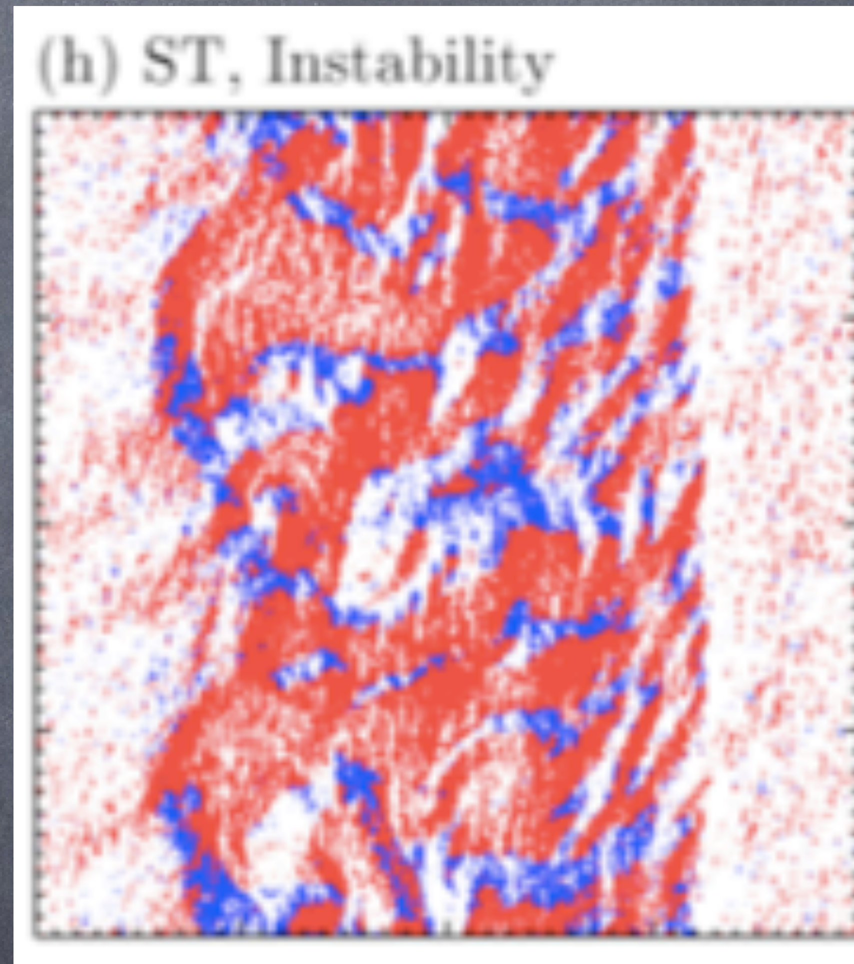


P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, G. P. Chini. Langmuir-Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spin-down Simulations, *JGR-Oceans*, 2013. In prep.

With Stokes Drift

Without Stokes Drift





Mostly Baroclinic & Symmetric & Gravitational Instability

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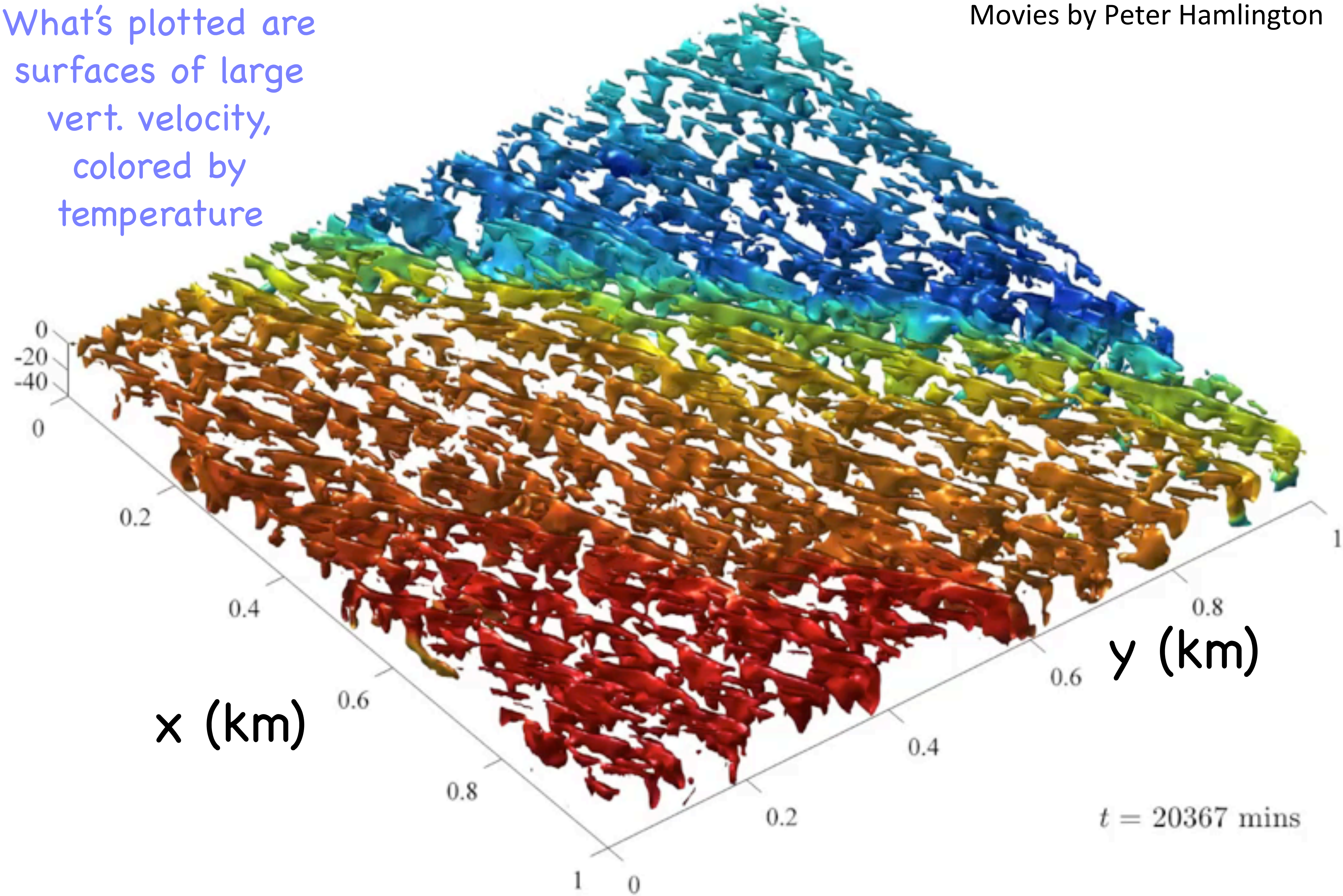
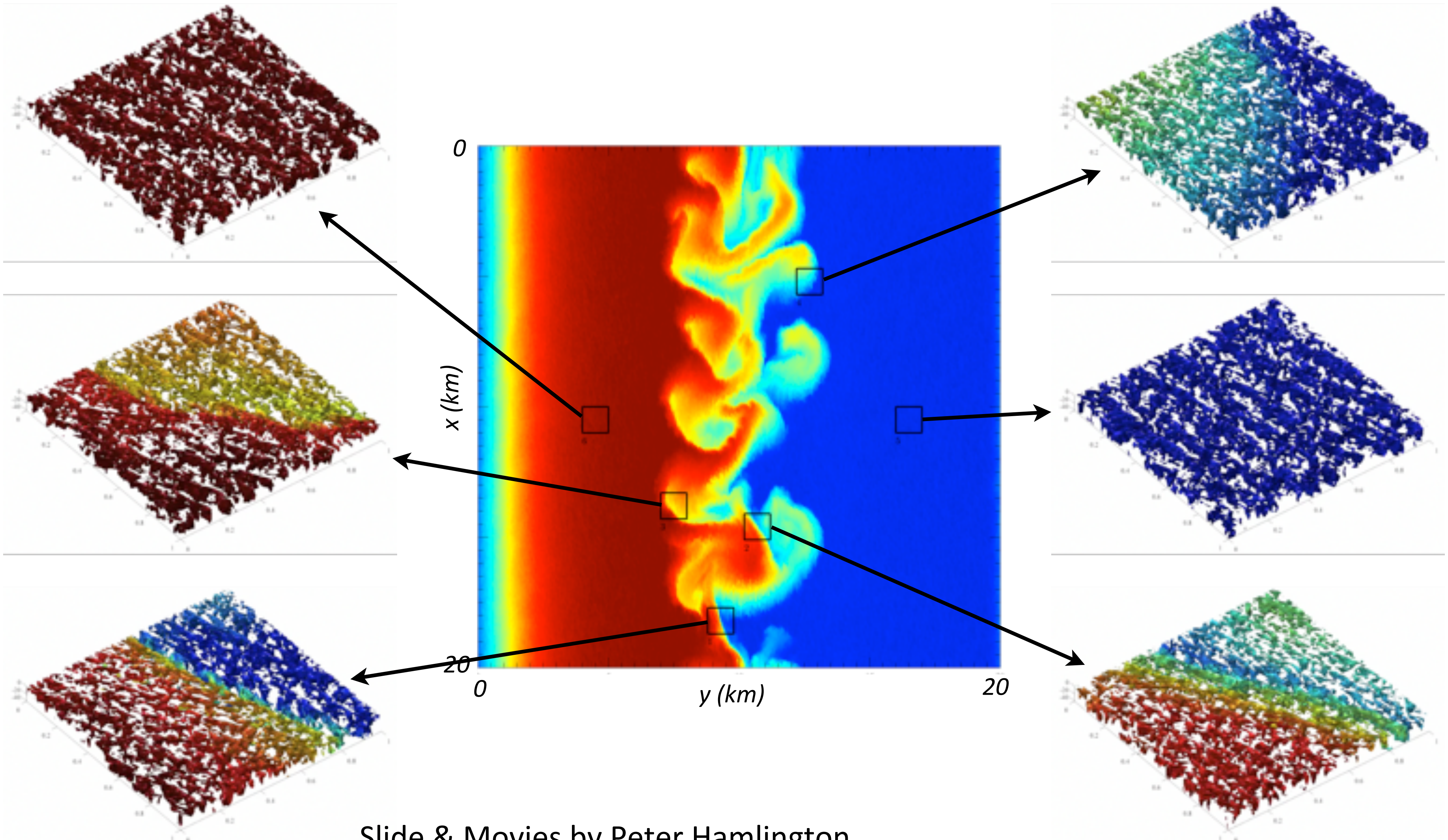
Zoom: Submeso-Langmuir Interaction!

What's plotted are surfaces of large vert. velocity, colored by temperature

Movies by Peter Hamlington



Diverse types of interaction

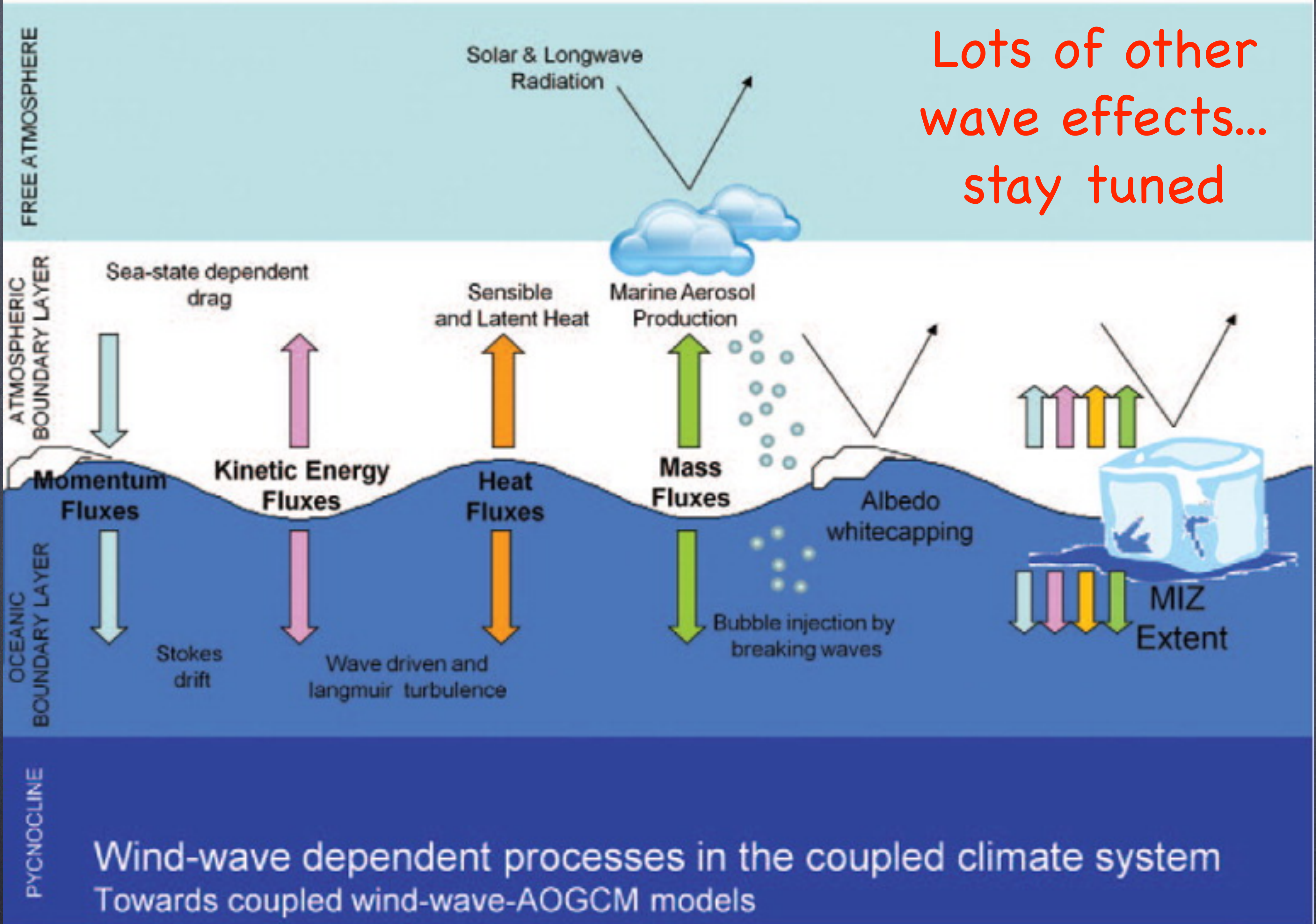


Slide & Movies by Peter Hamlington

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale simulations. In preparation, 2013.

Conclusions

- Waves are a dominant feature of the upper ocean on short timescales
- On longer timescales, rectified effects of waves--the Stokes drift--changes boundary layer turbulence and submesoscale dynamics
- Critical concept: Lagrangian shear takes over for Eulerian--except for a different *vortex force*
- Wave, convective, & wind effects are particularly important when transient
  - e.g., waves *not fully developed* which is most of the time for long fetch (i.e., open ocean)



L. Cavaleri, B. Fox-Kemper, and M. Hemer. Wind waves in the coupled climate system. Bulletin of the American Meteorological Society, 93(11):1651-1661, 2012.

Big Picture Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- In the upper ocean, horizontal scales as big as basins, and as small as centimeters contribute non-negligibly
- Process models are needed to study these connections and improve subgrid models.
- Interesting are the submeso to Langmuir scales, as nonhydro. & ageostrophic effects become dominant
- The CLB are good for LES & analysis in this range, but cannot capture some effects of small, steep waves (breaking, spray, nearshore, etc.)

So, no problems?
Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - CLB wave equations require limited *wave steepness* and irrotational flow
  - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...

Power Spectrum of wave height

$$\langle \eta^2 \rangle = \int_0^\infty E(k)dk = C_0 + \int_{k_h}^\infty C_1 k^{-2} dk$$

Power Spectrum of wave steepness: INFINITE!

$$\langle k^2 \eta^2 \rangle = \int_0^\infty k^2 E(k)dk = D_0 + \int_{k_h}^\infty D_1 dk$$

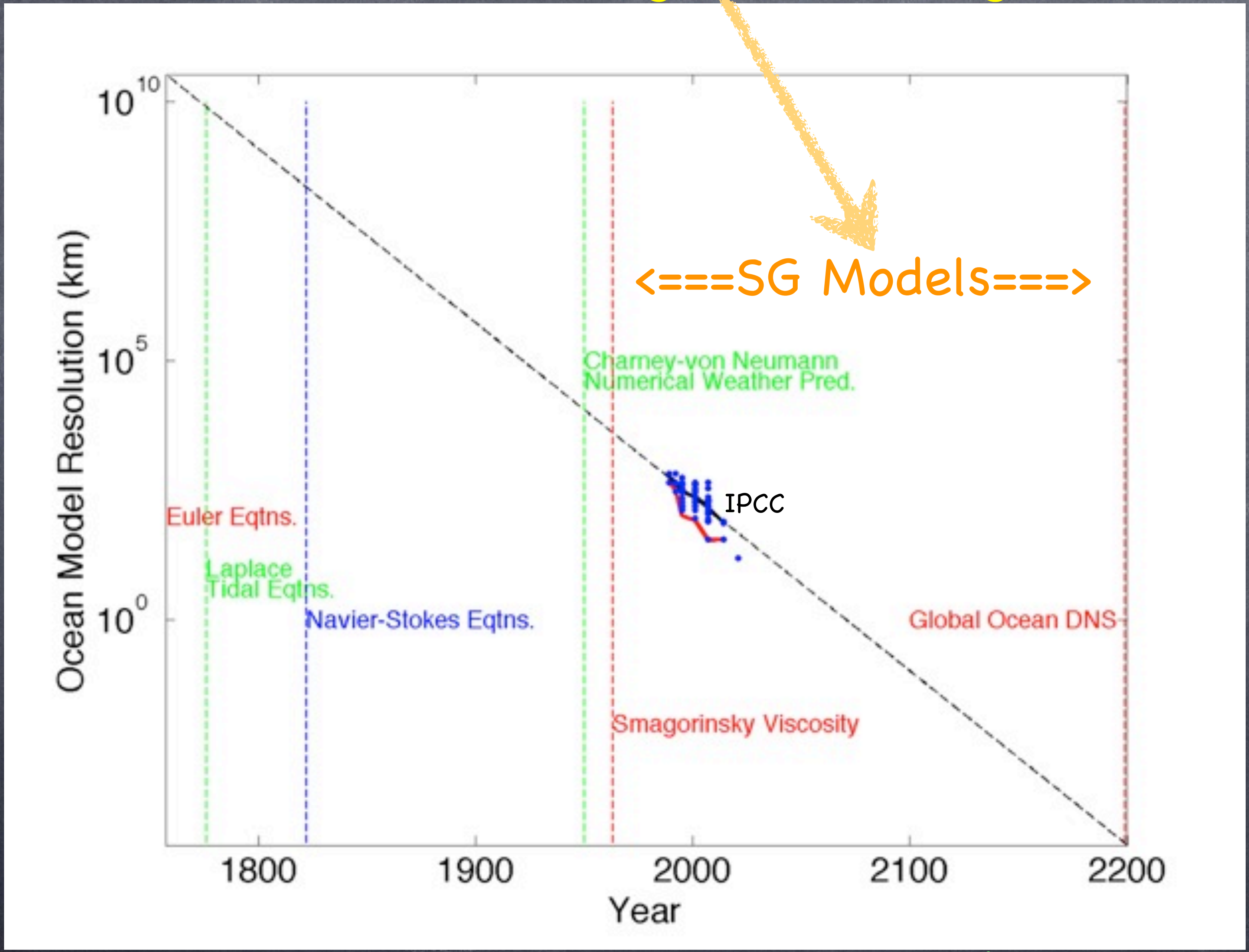
Steep waves break->vortex motion & small scale turbulence!

So, no problems?
Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - Also, what about finite wave packets?
  - What about co-evolution of the submesoscale flow and wave packets?
  - What about steep wave effects? Breaking?
  - Are there other ways for waves to drive turbulence?

Steep waves break->vortex motion & small scale turbulence!

Extrapolate for historical perspective: The Golden Era of Subgrid Modeling is Now!

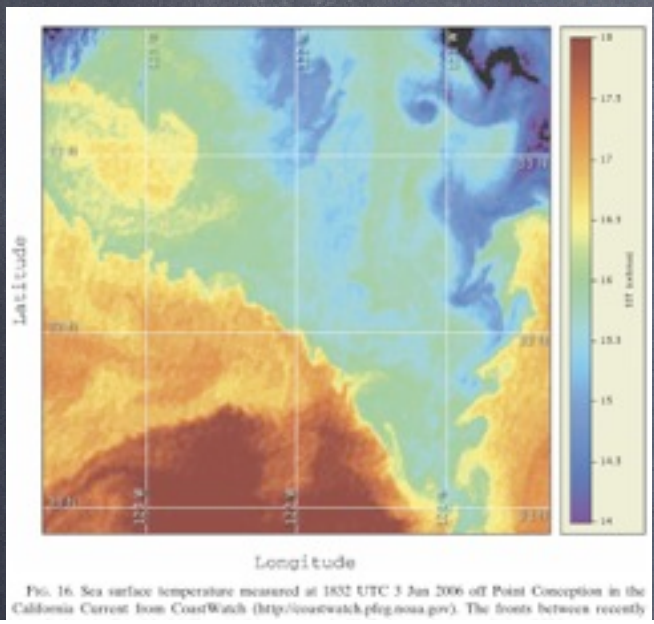


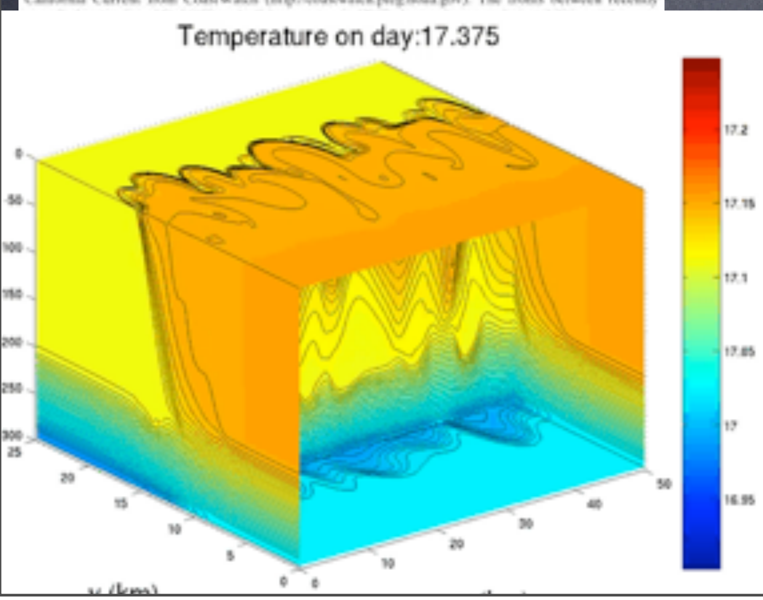
All papers at: fox-kemper.com/research

The Character of the Submesoscale

← 10 km

(Capet et al., 2008)



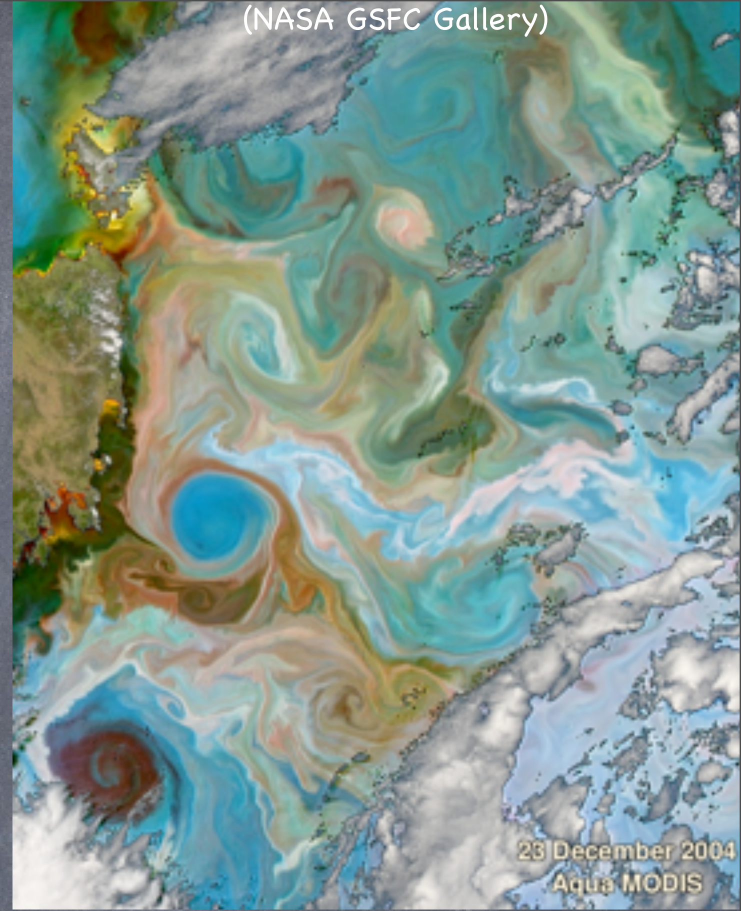


- Fronts
- Eddies
- Ro=O(1)
- Ri=O(1)
- near-surface
- 1-10km, days

Eddy processes often baroclinic instability

Parameterizations of submesoscale baroclinic instability?

(NASA GSFC Gallery)

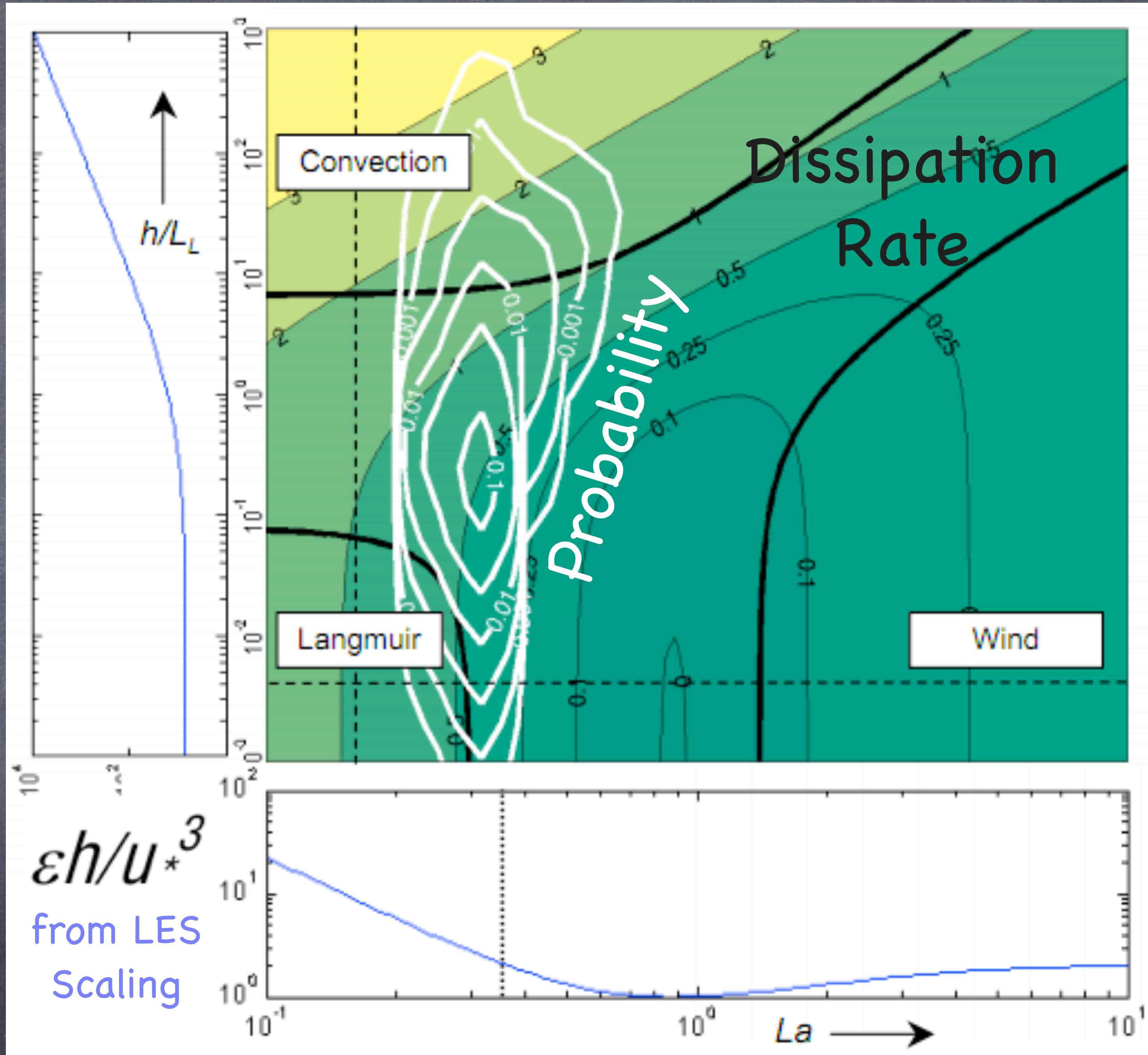


B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013

Data + LES, Southern Ocean mixing energy: Langmuir (Stokes-drift-driven) and Convective

So, waves can drive mixing via Stokes drift (combines with cooling & winds)



S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. Geophysical Research Letters, 39(18):L18605, 9pp, 2012.

Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

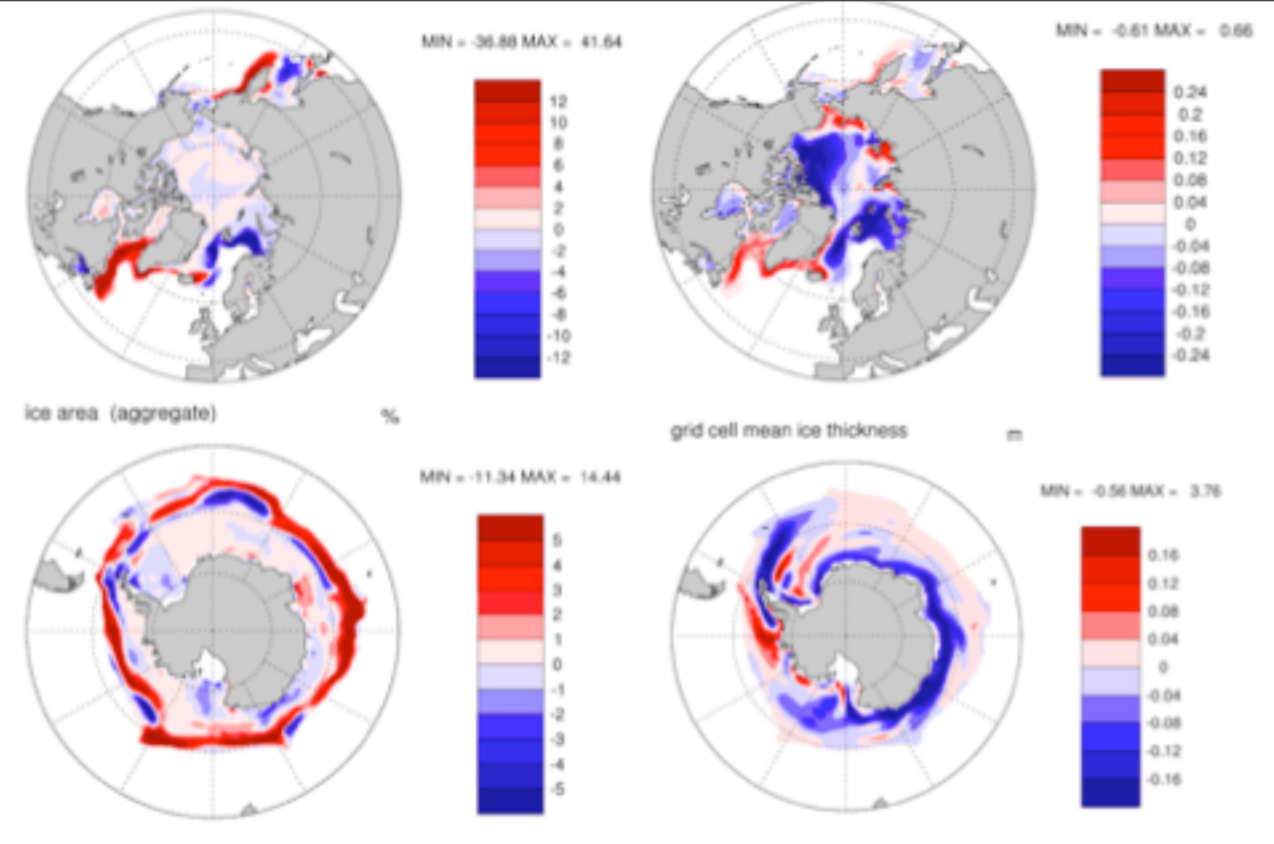
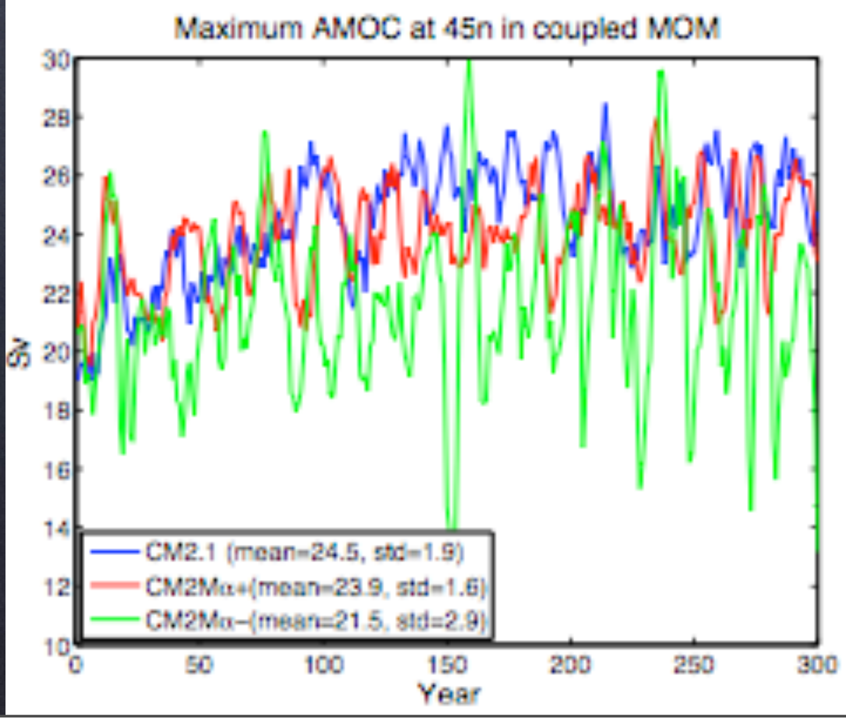


Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM$^+$ minus CCSM$^-$): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

May Stabilize AMOC



Affects sea ice

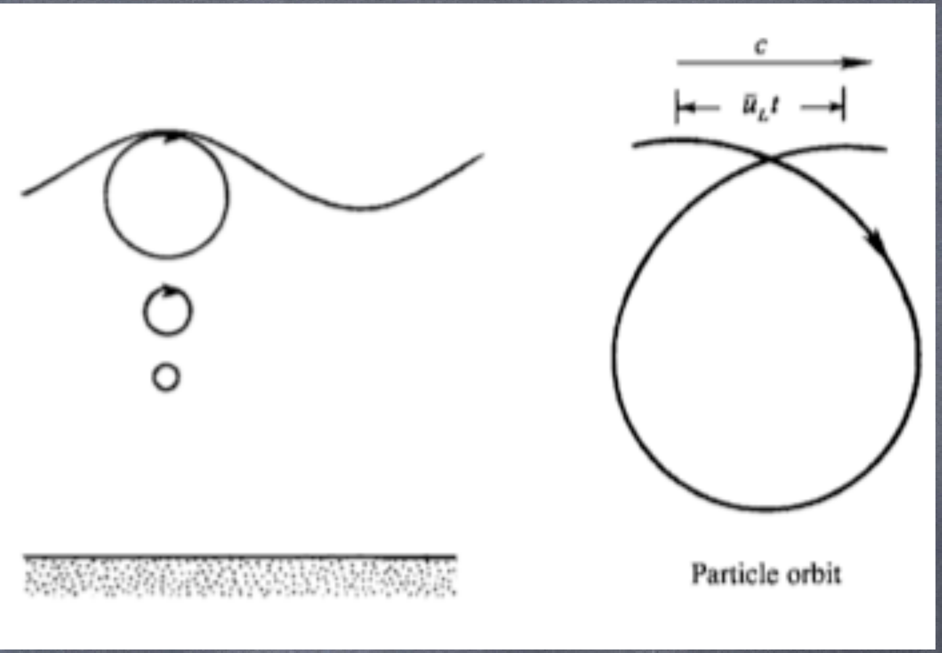
NO RETUNING NEEDED!!!

These are impacts:
bias change unknown

What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

$$\boldsymbol{u}^L(\boldsymbol{x}_p(t_0),t) - \boldsymbol{u}^E(\boldsymbol{x}_p(t_0),t) \approx [\boldsymbol{x}_p(t) - \boldsymbol{x}_p(t_0)] \cdot \nabla \boldsymbol{u}^E(\boldsymbol{x}_p(t_0),t)$$
$$\approx \left[\int_{t_0}^{t} \boldsymbol{u}^E(\boldsymbol{x}_p(t_0),s')ds'\right] \cdot \nabla \boldsymbol{u}^E(\boldsymbol{x}_p(t_0),t)\,.$$



Examples:

Monochromatic: $$\boldsymbol{u}^S = \hat{\mathbf{e}}^w \frac{8\pi^3 a^2 f_p^3}{g} e^{\frac{8\pi^2 f_p^2}{g}z} = \hat{\mathbf{e}}^w a^2 \sqrt{gk^3} e^{2kz}.$$

Spectrum: $$\boldsymbol{u}^S = \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^{\pi} (\cos\theta, \sin\theta, 0) f^3 \mathcal{S}_{f\theta}(f,\theta) e^{\frac{8\pi^2 f^2}{g}z} d\theta df.$$

A. Webb and B. Fox-Kemper. Wave spectral moments and Stokes drift estimation. *Ocean Modelling*, 40(3-4):273-288, 2011.

Craik-Leibovich Boussinesq

Old Boussinesq (written in vortex force form)

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times \mathbf{v} = -\nabla \pi + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + \mathbf{v} \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

Craik-Leibovich Boussinesq $\quad \mathbf{v}_s = \text{Stokes Drift}$

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^\dagger + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0$$

$$\nabla \cdot \mathbf{v} = 0$$

Global Picture: Misalignment enhances degree to which we expect wave-driven turbulence in Boundary layer

Wind-Driven

Wave-Driven

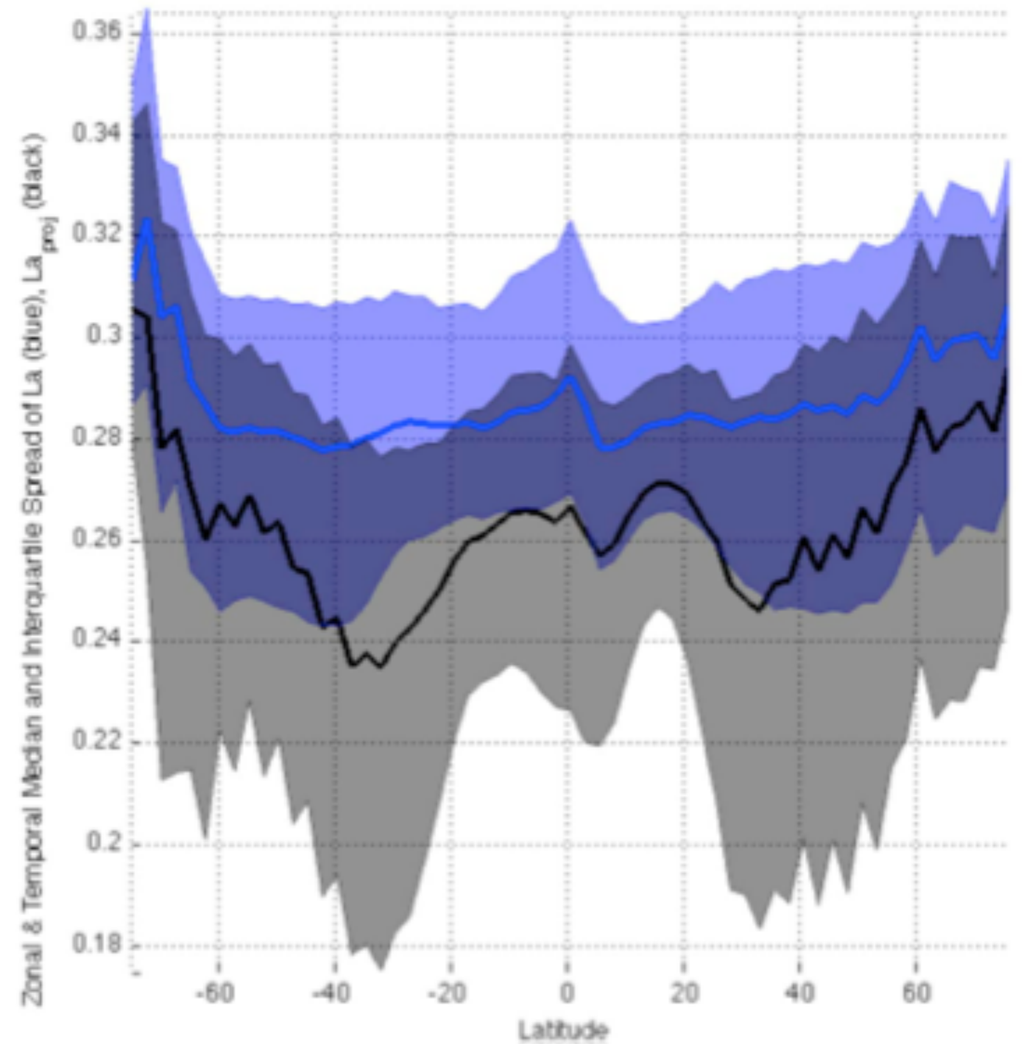


Figure 17. Temporal and zonal median and interquartile range of La_t and La_{proj} for a realistic simulation of 1994–2002 using Wave Watch III.